

CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- 1 radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

- π radian = 180 degree, $1^\circ = 60'$

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ = 57^\circ 16' 22'' \text{ (Appr.)}$$

- If an arc of length ' ℓ ' makes an angle ' θ ' radian at the centre of a circle of radius ' r ', we have $\theta = \frac{\ell}{r}$.

- | Quadrant → | I | II | III | IV |
|--------------------------------|-----|----------------|--------------|--------------|
| t-functions which are positive | All | sinx
cosecx | tanx
cotx | cosx
secx |

- | Function | Domain | Range |
|----------|---|---------------|
| sinx | R | [-1, 1] |
| cosx | R | [-1, 1] |
| tanx | $R - \left\{(2n+1)\frac{\pi}{2}\right\}; n \in Z$ | R |
| cosecx | $R - \{n\pi\}; n \in Z$ | $R - (-1, 1)$ |
| secx | $R - \left\{(2n+1)\frac{\pi}{2}\right\}; n \in Z$ | $R - (-1, 1)$ |
| cotx | $R - \{n\pi\}; n \in Z$ | R |

- **Allied or related angles:** The angles $\frac{n\pi}{2} \pm \theta$ are called allied or related angles and $\theta \pm n \times 360^\circ$ are called conterminal angles. For general reduction we have the following rules. The value of any trigonometric functions for $\left(\frac{n\pi}{2} \pm \theta\right)$ is numerically equal to :-
 - (a) The value of the same function if n is even integer with algebraic sign of the function as per the quadrant in which angles π .
 - (b) Corresponding co-function of ' θ ' if n is an odd integer algebraic sign of the function for the quadrant in which it lies. Here sine and cosine; tan and cot; sec and cosec, are co-functions of each other.

- **Trigonometric Identities:**
 - (i) $\sin(x + y) = \sin x \cos y + \cos x \sin y$
 - (ii) $\sin(x - y) = \sin x \cos y - \cos x \sin y$
 - (iii) $\cos(x + y) = \cos x \cos y - \sin x \sin y$
 - (iv) $\cos(x - y) = \cos x \cos y + \sin x \sin y$
 - (v) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$
 - (vi) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$
 - (vii) $\cot(x + y) = \frac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$
 - (viii) $\cot(x - y) = \frac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$

$$(ix) \quad \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$(x) \quad \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$(xi) \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$(xii) \quad \sin 3x = 3 \sin x - 4 \sin^3 x$$

$$(xiii) \quad \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$(xiv) \quad \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$$(xv) \quad \cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$(xvi) \quad \cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}$$

$$(xvii) \quad \sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$(xviii) \quad \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$(xix) \quad 2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$(xx) \quad 2 \cos x \sin y = \sin(x+y) - \sin(x-y)$$

$$(xxi) \quad 2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

$$(xxii) \quad 2 \sin x \sin y = \cos(x-y) - \cos(x+y)$$

$$(xxiii) \quad \sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$(xxiv) \quad \cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$(xxv) \quad \tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Sing '+' or '-' will be decided according to the quadrant in which angle $\frac{A}{2}$ lies.

- **Solution of trigonometric equations:**
 - (a) **Principal Solutions:** The solutions of a trigonometric equation for which $0 \leq \theta \leq 2\pi$ are called principal solutions.
 - (b) **General Solutions:** The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution.

- **General solution of trigonometric equations:**
 - (i) If $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
 - (ii) If $\cos\theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}$
 - (iii) If $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
 - (iv) If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha, n \in \mathbb{Z}$
 - (v) If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$
 - (vi) If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, n \in \mathbb{Z}$
 - (vii) If $\sin^2\theta = \sin^2\alpha, \Rightarrow \theta = n\pi \pm \alpha, n \in \mathbb{Z}$
 $\cos^2\theta = \cos^2\alpha,$
 $\tan^2\theta = \tan^2\alpha$

- Maximum and minimum values of the expression $A\cos\theta + B\sin\theta$ are $\sqrt{A^2 + B^2}$ and $-\sqrt{A^2 + B^2}$ respectively, where A and B are constants.

- $\sin 18^\circ = \frac{\sqrt{5}-1}{4} \qquad \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$
 $\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4} \qquad \cos 36^\circ = \frac{\sqrt{5}+1}{4}$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the radian measure of $5^{\circ} 37' 30''$.
2. Write the degree measure of $\frac{11}{16}$ radian.
3. Write the value of $\tan\left(\frac{19\pi}{3}\right)$.
4. What is the value of $\sin(-1125^{\circ})$.
5. Write the general solution of $\sin\left(x + \frac{\pi}{12}\right) = 0$.
6. Write the value of $2\sin 75^{\circ} \sin 15^{\circ}$.
7. What is the maximum value of $3 - 7 \cos 5x$.
8. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.
9. Express $2\cos 4x \sin 2x$ as an algebraic sum of sines and cosines.
10. Write the maximum value of $\cos(x)$ and also write its minimum value.
11. Write is the value of $\tan\frac{\pi}{12}$.

Choose the correct answer from the given four options in exercise 12 to 30.

12. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is -
- | | |
|--|-------------------------------------|
| (a) $\frac{-4}{5}$ but not $\frac{4}{5}$ | (b) $\frac{-4}{5}$ or $\frac{4}{5}$ |
| (c) $\frac{4}{5}$ but not $\frac{-4}{5}$ | (d) None of these. |

13. The greatest value of $\sin x \cos x$ is -
(a) 1 (b) 2
(c) $\sqrt{2}$ (d) $\frac{1}{2}$.
14. If $\sin \theta + \operatorname{cosec} \theta = 2$, then $\sin^2 \theta + \operatorname{cosec}^2 \theta$ is equal to -
(a) 1 (b) 4
(c) 2 (d) None of these.
15. If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$ then the value of $\theta + \phi$ is -
(a) $\frac{\pi}{6}$ (b) π
(c) 0 (d) $\frac{\pi}{4}$.
16. Which of the following is not correct -
(a) $\sin \theta = \frac{-1}{5}$ (b) $\cos \theta = 1$
(c) $\sec \theta = \frac{1}{2}$ (d) $\tan \theta = 20$.
17. The value of $\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \dots \dots \dots \tan 89^\circ$ is -
(a) 0 (b) 1
(c) $\frac{1}{2}$ (d) Not defined.
18. The value of $\cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \dots \dots \dots \cos 179^\circ$ is -
(a) $\frac{1}{\sqrt{2}}$ (b) 0
(c) 1 (d) -1.

19. The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is -
- (a) 1 (b) $\sqrt{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) 2.
20. The minimum value of $3\cos x + 4\sin x + 8$ is -
- (a) 5 (b) 9
(c) 7 (d) 3.
21. The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is equal to -
- (a) 1 (b) 0
(c) $\frac{1}{2}$ (d) 2.
22. If $\sin \theta + \cos \theta = 1$, then the value of $\sin 2\theta$ is equal to -
- (a) 1 (b) $\frac{1}{2}$
(c) 0 (d) 2.
23. If $\alpha + \beta = \frac{\pi}{4}$, then value of $(1 + \tan \alpha) \cdot (1 + \tan \beta)$ is -
- (a) 1 (b) 2
(c) -2 (d) Not defined.
24. The value of $\cos^2 48^\circ - \sin^2 12^\circ$ is -
- (a) $\frac{\sqrt{5}+1}{8}$ (b) $\frac{\sqrt{5}-1}{8}$
(c) $\frac{\sqrt{5}+1}{5}$ (d) $\frac{\sqrt{5}+1}{2\sqrt{2}}$.

25. The number of solutions of the equation $4\sin x - 3\cos x = 7$ are -
(a) 0 (b) 1
(c) 2 (d) 3.
26. If $\cos x = \frac{1}{2}\left(a + \frac{1}{a}\right)$, then $\cos 3x$ is -
(a) $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ (b) $\frac{3}{2}\left(a^3 + \frac{1}{a^3}\right)$
(c) $\frac{1}{2}\left(a^3 - \frac{1}{a^3}\right)$ (d) $\frac{3}{2}\left(a^3 - \frac{1}{a^3}\right)$.
27. If $\cos x = \frac{-1}{2}$ and $0 < x < 2\pi$, then solutions are -
(a) $x = \frac{\pi}{3}, \frac{4\pi}{3}$ (b) $x = \frac{2\pi}{3}, \frac{4\pi}{3}$
(c) $x = \frac{2\pi}{3}, \frac{7\pi}{6}$ (d) $x = \frac{2\pi}{3}, \frac{\pi}{3}$.
28. If $P = 2\sin^2 x - \cos^2 x$, then P lies in the interval -
(a) [1, 3] (b) [1, 2]
(c) [-1, 2] (d) None of these.
29. If $\frac{\pi}{4} < x < \frac{\pi}{2}$, then write the value of $\sqrt{1 - \sin 2x}$ is -
(a) $\cos x - \sin x$ (b) $\cos x + \sin x$
(c) $\sin x - \cos x$ (d) 2.
30. If $\sin x + \cos x = a$, then the value of $|\sin x - \cos x|$ is -
(a) $\sqrt{2 - a^2}$ (b) $\sqrt{a^2 - 2}$
(c) $\sqrt{a^2 + 2}$ (d) 1.

Fill in the blanks (Exercise 31 to 35) :-

31. The value of $\frac{\sin 50^\circ}{\sin 130^\circ}$ is _____.
32. If $\tan A = \frac{1 - \cos B}{\sin B}$, $\tan 2A =$ _____.
33. If $3\sin x + 4\cos x = 5$, then $4\sin x - 3\cos x$ is _____.
34. If $\cos(A - B) = \frac{3}{5}$ and $\tan A \tan B = 2$, then the value of $\cos A \cos B$ is _____.
35. If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$, then the value of $\cos\left(\frac{A - B}{2}\right)$ is _____.
36. If the following match each item given under column C_1 to its correct answer given under column C_2 :-

C_1	C_2
(a) $\sin(x + y) \sin(x - y)$	(i) $\cos^2 x - \sin^2 y$
(b) $\cos(x + y) \cos(x - y)$	(ii) $\frac{1 - \tan x}{1 + \tan x}$
(c) $\cot\left(\frac{\pi}{4} + x\right)$	(iii) $\frac{1 + \tan x}{1 - \tan x}$
(d) $\tan\left(\frac{\pi}{4} + x\right)$	(iv) $\sin^2 x - \sin^2 y$

37. Match each item given under column C_1 to its correct answer given under column C_2 :-

C_1	C_2
(a) $\frac{1 - \cos x}{\sin x}$	(i) $\cot^2 \frac{x}{2}$
(b) $\frac{1 + \cos x}{1 - \cos x}$	(ii) $\cot \frac{x}{2}$
(c) $\frac{1 + \cos x}{\sin x}$	(iii) $ \cos x + \sin x $
(d) $\sqrt{1 + \sin 2x}$	(iv) $\sin^2 x - \sin^2 y$

The statements given are true or false (Exercise 38 to 45) :-

38. If $0 \leq x \leq \pi$ then $\cos 0 \leq \cos x \leq \cos \pi$.
39. If $0 \leq x \leq \frac{\pi}{2}$ then $\sin 0 \leq \sin x \leq \sin \frac{\pi}{2}$.
40. If $\pi \leq x \leq \frac{3\pi}{2}$ then $\sin \pi \leq \sin x \leq \sin \frac{3\pi}{2}$.
41. If $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$ then $\tan x$ is an increasing function.
42. The period of $\sin x$ function is ' 2π '
43. The period of $\cos x$ function is ' 2π '
44. The period of $\tan x$ function is ' 2π '
45. The range of $f(x) = \sec x$ is $\mathbb{R} - [-1, 1]$.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

46. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15° .
47. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$ Find $\cos A$, $\sin 2A$
48. What is the sign of $\cos x/2 - \sin x/2$ when
(i) $0 < x < \pi/4$ (ii) $\frac{\pi}{2} < x < \pi$
49. Prove that $\cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ = -1$
50. Find the maximum and minimum value of $7 \cos x + 24 \sin x$
51. Evaluate $\sin(\pi + x) \sin(\pi - x) \operatorname{cosec}^2 x$
52. Find the angle in radians between the hands of a clock at 7 : 20 PM.
53. If $\cot \alpha = \frac{1}{2}$ $\sec \beta = \frac{-5}{3}$ where $\pi < \alpha < 3\pi/2$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of $\tan(\alpha + \beta)$
54. If $\cos x = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$. Find the value of $\cos x/2$, $\tan x/2$
55. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

56. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.
57. Find the minimum and maximum value of
 $\sin^4 x + \cos^2 x; \quad x \in R.$

58. Solve $\sec x \cdot \cos 5x + 1 = 0$
59. Solve $2 \tan^2 x + \sec^2 x = 2$ for $0 < x^2 \pi$.
60. Solve $\sqrt{3} \cos x - \sin x = 1$.
61. Solve $\sqrt{2} \sec \theta - \tan \theta = \sqrt{3}$.
62. Solve $3 \tan x + \cot x = 5 \operatorname{cosec} x$.
63. Find x if $3 \tan (x - 15^\circ) = \tan (x + 15^\circ)$
64. Solve $\tan x + \tan 2x + \sqrt{3} \tan x \cdot \tan 2x = \sqrt{3}$.
65. Solve $\tan x + \sec x = \sqrt{3}$.
66. If $\sec x = \sqrt{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find the value of $\frac{1 - \tan x - \operatorname{cosec} x}{1 - \cot x - \operatorname{cosec} x}$.
67. Prove that $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$.
68. If $f(x) = \frac{\cot x}{1 + \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$ then find $f(\alpha) \cdot f(\beta)$.
69. Prove that $\tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ$
70. Prove that $\tan 13x = \tan 4x + \tan 9x + \tan 4x \tan 9x \tan 13x$.

Prove the following Identities

71.
$$\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cdot \cos 4\theta.$$
72.
$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x.$$

$$73. \frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \cdot \sin x + \cos 6x \cdot \cos x} = \tan 2x.$$

$$74. \frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}.$$

$$75. \tan \alpha \cdot \tan(60^\circ - \alpha) \cdot \tan(60^\circ + \alpha) = \tan 3\alpha.$$

$$76. \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = 2 \cos \theta.$$

$$77. \frac{\cos x}{1 - \sin x} = \tan \left(\frac{\pi}{4} + \frac{x}{2} \right).$$

$$78. \cos 10^\circ + \cos 110^\circ + \cos 130^\circ = 0.$$

$$79. \frac{\sin(x+y) - 2 \sin x + \sin(x-y)}{\cos(x+y) - 2 \cos x + \cos(x-y)} = \tan x$$

$$80. \sin x + \sin 2x + \sin 4x + \sin 5x = 4 \cos \frac{x}{2} \cdot \cos \frac{3x}{2} \cdot \sin 3x$$

$$81. \frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$$

$$82. \text{Find the value of } \sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$$

$$83. \cos \frac{\pi}{5} \cdot \cos \frac{2\pi}{5} \cdot \cos \frac{4\pi}{5} \cdot \cos \frac{8\pi}{5} = \frac{1}{16}$$

$$84. \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ = \frac{1}{8}$$

98. If α and β are the solution of the equation, $a \tan \theta + b \sec \theta = c$ then show that $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$.

99. Prove that $\cos^2 x + \cos^2 y - 2 \cos x \cdot \cos y \cdot \cos(x + y) = \sin^2(x + y)$

100. Prove that $2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta) = \cos 2\alpha$

101. Solve $81^{\sin^2 x} + 81^{\cos^2 x} = 30 \quad 0 < x < \pi$

102. Find the minimum value of p for which $\cos(p \sin x) = \sin(p \cos x)$ has a solution in $[0, 2\pi]$.

103. Prove that $\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \cdot \sin A}$.

104. Solve: $4 \sin x \cdot \sin 2x \cdot \sin 4x = \sin 3x$

105. Solve: $\cos \theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

106. Evaluate: $\left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right)$

107. Prove that $4 \sin \alpha \cdot \sin \left(\alpha + \frac{\pi}{3}\right) \cdot \sin \left(\alpha + \frac{2\pi}{3}\right) = \sin 3\alpha$.

ANSWERS

1. $\frac{\pi}{32}$
2. $39^\circ 22' 30''$
3. $\sqrt{3}$
4. $-\frac{1}{\sqrt{2}}$
5. $n\pi - \frac{\pi}{12}, n \in \mathbb{Z}$.
6. $\frac{1}{2}$
7. 10
8. $2 \sin 8\theta \cos 4\theta$
9. $\sin 6x - \sin 2x$
10. 1 and -1
11. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
12. (b)
13. (d)
14. (c)
15. (d)
16. (c)
17. (b)
18. (b)
19. (c)
20. (d)
21. (b)
22. (c)
23. (b)
24. (a)
25. (a)
26. (a)
27. (b)
28. (c)
29. (c)
30. (a)
31. 1
32. $\tan B$
33. 0
34. $\frac{1}{5}$
35. $\frac{1}{\sqrt{3}}$
36. (a) \rightarrow (iv)
(b) \rightarrow (i)
(c) \rightarrow (ii)
(d) \rightarrow (iii)
37. (a) \rightarrow (iv)
(b) \rightarrow (i)
(c) \rightarrow (ii)
(d) \rightarrow (ii)

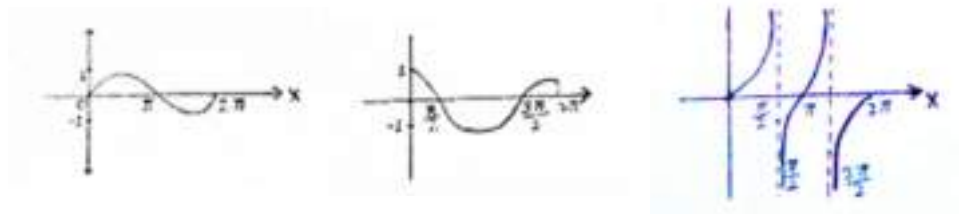
38. False
39. True
40. False
41. True
42. True
43. True
44. False
45. False
46. 70m
47. $\frac{-4}{5}, \frac{-24}{25}$
48. (i) + ve (ii) -ve
50. Max value 25;
Min value -25
51. -1
52. $\frac{5\pi}{9}$
53. $\frac{2}{11}$
54. $-1/\sqrt{3}, -2$
55. $\pi/4$
56. 70 m
57. $\min = \frac{3}{4}, \max = 1$
58. $x = (2n+1)\frac{\pi}{6}$, or $x = (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}$
59. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
60. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, n \in \mathbb{Z}$
61. $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}, n \in \mathbb{Z}$
62. $2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$
63. $x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}, n \in \mathbb{Z}$
64. $\frac{n\pi}{3} + \frac{\pi}{9}$
65. $2n\pi \pm \frac{2\pi}{3} - \frac{2\pi}{6} - \frac{\pi}{6}, n \in \mathbb{Z}$
66. 1
68. $\frac{1}{2}$
82. 4

85. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, n \in \mathbb{Z}$

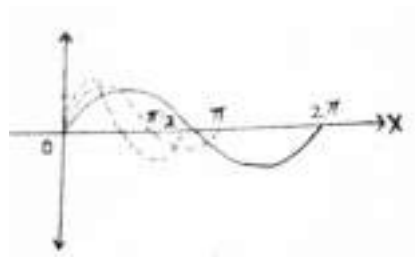
86. $\frac{1}{5}(2n\pi + \frac{\pi}{2}), 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

87. $x = \frac{n\pi}{2}, n \in \mathbb{Z}$

88.



89.



90. (i) $\frac{1+\sqrt{5}}{4}$ (ii) $2-\sqrt{3}$

91. $\frac{3}{2}$

97. $[-6, 20]$

101. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

102. $\frac{\pi}{\sqrt{8}}, \frac{5\sqrt{2}\pi}{4}$

104. $n\pi, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$

105. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$

106. $\frac{1}{8}$