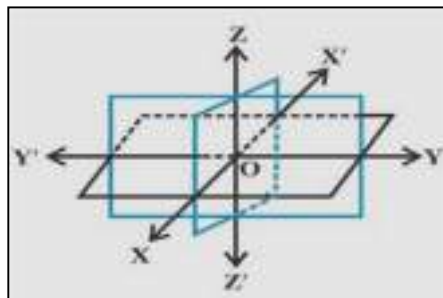
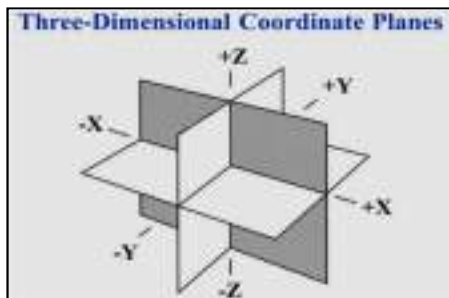


CHAPTER - 12

INTRODUCTION TO THREE-DIMENSIONAL COORDINATE GEOMETRY

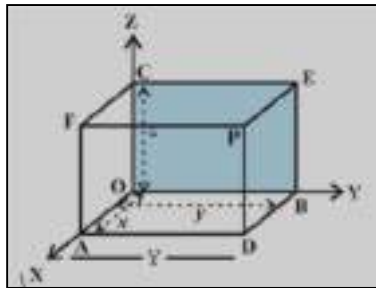
KEY POINTS

- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.



- ❖ **Coordinate axes:** XOX' , YOY' , ZOZ'
- ❖ **Coordinate planes:** XOY , YOZ , ZOX or XY , YX , ZX planes
- ❖ **Octants:** $OXYZ$, $OX'YZ$, $OXY'Z$, $OXYZ'$, $OX'Y'Z$, $OXY'Z'$, $OX'YZ'$, $OX'Y'Z'$
- ❖ Coordinates of a points lying on x-axis, y-axis and z-axis are of the form $(x, 0, 0)$, $(0, y, 0)$, $(0, 0, z)$ respectively.
- ❖ Coordinates of a points lying on xy-plane, yz-plane and xz-plane are of the form $(x, y, 0)$, $(0, y, z)$, $(x, 0, z)$ respectively.

- ❖ The reflection of the point (x, y, z) in xy -plane, yz -plane and xz -plane is $(x, y, -z)$, $(-x, y, z)$ and $(x, -y, z)$ respectively.
- ❖ Absolute value of the Coordinates of a point $P(x, y, z)$ represents the perpendicular distances of point P from three coordinate planes YZ , ZX and XY respectively.



- The distance between the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio $m : n$

- (a) Internally, then the coordinates of R are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right).$$

- (b) Externally, then the coordinates of R are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right).$$

- Coordinates of Centroid of a triangle whose vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right).$$

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Fill up in each of the following:
 - (a) The coordinate of the image of $(-1, 2, -3)$ in x-z plane is _____.
 - (b) The coordinate of the image of $(-1, 2, -3)$ in y-z plane is _____.
 - (c) The coordinate of the image of $(-1, 2, -3)$ in x-y plane is _____.
 - (d) The Point P $(-5, 4, -3)$ lies in the octant _____.
 - (e) If $a < 0$, $b > 0$ & $c < 0$, then The Point P $(a, b, -c)$ lies in the octant _____.
 - (f) The perpendicular distance of the point P $(-6, 7, -8)$ from xy-plane is _____.
 - (g) The perpendicular distance of the point P $(-3, 5, -12)$ from x-axis is _____.
 - (h) The perpendicular distance of the point P $(-3, 4, -5)$ from z-axis is _____.
 - (i) The coordinates of foot of perpendicular from $(3, 7, 9)$ on y-axis is _____.

 2. State whether the following statements are true or false.
 - (a) If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then the sum of all possible value of a is 2.
 - (b) The x-axis and z-axis, together determine a plane known as yz-plane.
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- (c) The point P (1, 2, -3) lies in the 7th octant.
- (d) The y-axis is the intersection of two planes xy-plane and yz-plane.
- (e) Distance of the point (3, 4, 5) from the origin (0, 0, 0) is 10.
- (f) The distance of point P(-3, -4, -5) from the yz-plane is -3.
- (g) The distance of point P(-3, -4, -5) from the y-axis is -4.
- (h) If $(c - 1) > 0$, $(a + 2) < 0$ and $b > 0$ then the point P (a, -b, c) lies in the 4th octant.
- (i) The length of the foot of perpendicular drawn from the point P (5, 12, 10) on z-axis is 10.

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

11. What are the coordinates of the vertices of a cube whose edge is 2 unit, one of whose vertices coincides with the origin and the three edges passing through the origin?
Coincides with the positive direction of the axes through the origin?
12. Let A, B, C be the feet of perpendiculars from point P(1, -2, -3) on the xy-plane, yz-plane and xz-plane respectively. Find the coordinates of A, B, C.
13. If a parallelepiped is formed by planes drawn through the point (5, 8, 10) and (3, 6, 8) parallel to the coordinates planes, then find the length of the gonol of the parallelepiped.
14. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13, 10 and 8 unit.

15. Find the coordinate of the point P which is three-fourth of the way from A(-1, 0, 2) to B (5, -7, -10).

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. Show that points (4, -3, -1), (5, -7, 6) and (3, 1, -8) are collinear.
17. Find the point on y-axis which is equidistant from the point (3, 1, 2) and (5, 5, 2).
18. Determine the point in yz plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).
19. The centroid of $\triangle ABC$ is at (1, 1, 1). If coordinates of A and B are (3, -5, 7) and (-1, 7, -6) respectively, find coordinates of point C.
20. Find the length of the medians of the triangle with vertices A(0, 0, 3), B(0, 4, 0) and C (5, 0, 0).
21. If the extremities (end points) of a diagonal of a square are (1, -2, 3) and (2, -3, 5) then find the length of the side of square.
22. Three consecutive vertices of a parallelogram ABCD are A(6, -2, 4) B(2, 4, -8), C(-2, 2, 4). Find the coordinates of the fourth vertex.
23. If the points A(1, 0, -6), B(3, p, q) and C(5, 9, 6) are collinear, find the value of p and q.
24. Show that the point A(1, 3, 0), B(-5, 5, 2), C(-9, -1, 2) and D(-3, -3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle.

25. The mid-points of the sides of a triangle are $(5, 7, 11)$, $(0, 8, 5)$ and $(2, 3, -1)$. Find its vertices and hence find centroid.
26. Find the coordinate of the points which divides the line segment AB in four equal parts where $A(-2, 0, 6)$ and $B(10, -6, -12)$.
27. Prove that the points $(0, -1, -7)$, $(2, 1, -9)$ and $(6, 5, -13)$ are collinear. Find the ratio in which first point divides the join of the other two.
28. Let $A(3, 2, 0)$, $B(5, 3, 2)$ $C(-9, 6, -3)$ be three points forming a triangle. AD, the bisector of $\angle BAC$, meets BC in D. Find the coordinates of the point D.
29. Describe the vertices and edges of the rectangular parallelepiped with one vertex $(3, 4, 5)$ placed in the first octant with one vertex at origin and edges of parallelepiped lie along x, y and z-axis.
30. Find the coordinates of the point which is equidistant from the point $(3, 2, 2)$, $(-1, 2, 2)$, $(0, 5, 6)$ and $(2, 1, 2)$.