CHAPTER - 10

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B (x_2, y_2)$ is given by

  $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

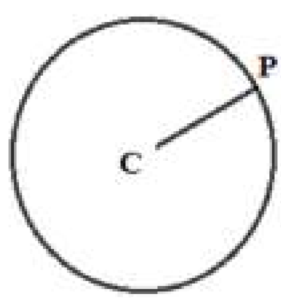
- Let the vertices of a triangle ABC are $A(x_1, y_1)$ $B (x_2, y_2)$ and $C(x_3, y_3)$.Then area of triangle

  $$ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

  Note: Area of a triangle is always positive. If the above expression is zero, then a triangle is not possible. Thus the points are collinear.

- **LOCUS:** When a variable point P(x, y) moves under certain condition then the path traced out by the point P is called the locus of the point.

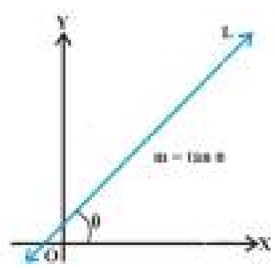
  For example: Locus of a point P, which moves such that its distance from a fixed point C is always constant, is a circle.



CP = constant

- Locus of an equation: In the coordinate plane, locus of an equation is the pictorial representation of the set of all those points which satisfy the given equation.
- Equation of a locus: is the equation in x and y that is satisfied by the coordinates of every point on the locus.
- A line is also defined as the locus of a point satisfying the condition $ax + by + c = 0$ where a, b, c are constants.
- **Slope of a straight line:**

  If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m

  $m = \tan\theta$, $\theta \neq 90°$

  If $0° < \theta < 90°$ then $m > 0$ and

  $90° < \theta < 1800°$ then $m < 0$

  Note-1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined

  Note-2: Slope of any horizontal line i.e. || to x-axis is zero. Slope of a vertical line i.e. || to y-axis is not zero.

- Three points A, B and C lying in a plane are collinear, if slope of AB = Slope of BC.
- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

- Two lines are parallel to each other if and only if their slopes are equal.

  i.e., $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$.

  Note: If slopes of lines l_1 and l_2 are not defined then they must be $\perp$ to x-axis, so they are ||. Thus $l_1 \parallel l_2 \Leftrightarrow$ they have same slope or both of them have not define slopes.

- Two non- vertical lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

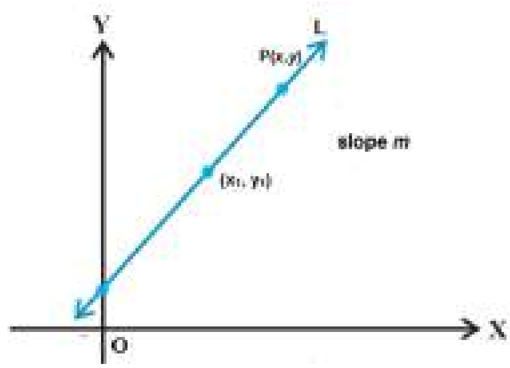
  i.e., $l_1 \perp l_2 \Leftrightarrow m_1\, m_2 = -1 \Leftrightarrow m_2 = \frac{-1}{m}$.

  Note: The above condition holds when the lines have non-zero slopes i.e none of them $\perp$ to any axis.

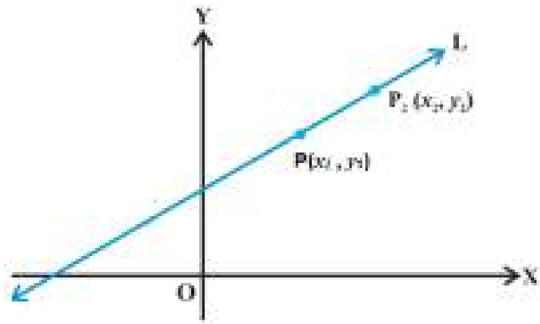
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$ and obtuse angle is $\phi = 180 - \alpha$.

- **Point slope form:**

  Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$



- **Two Point Form:**



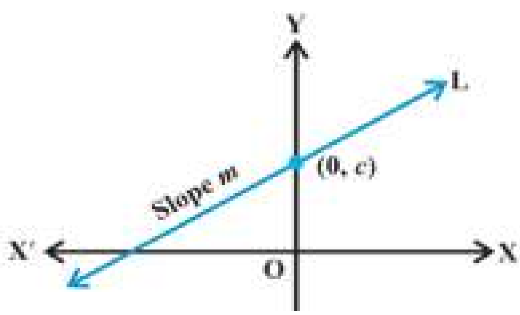
Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

- **Slope intercept form (y-intercept):**

Equation of a line having slope m and y-intercept 'c' is given by

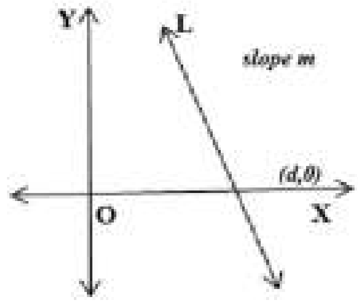
$y = mx + c$



- **Slope intercept form (x-intercept):**

Equation of a line having slope m and y-intercept c is given by

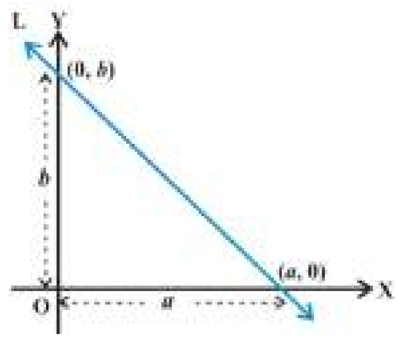
$y = m (x - d)$



- **Intercept Form:**

  Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

  $$\frac{x}{a}+\frac{y}{b}=1$$

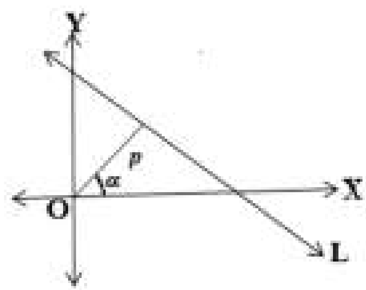


- **Normal Form:**

  Equation of line in normal form is given by x cosα + y sinα = p,

  p = Length of perpendicular segment from origin to the line

  α = Angle which the perpendicular segment makes with positive direction of x-axis



- **General Equation of a line:**

  Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non-zero.

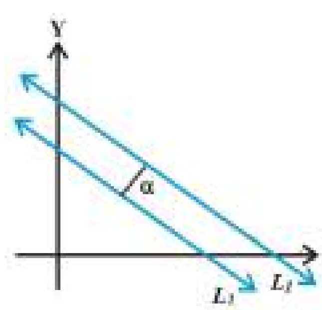
  Slope = $\frac{-A}{B}$ and y-intercept = $\frac{-C}{B}$ x-intercept = $\frac{-C}{A}$.

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

  $$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

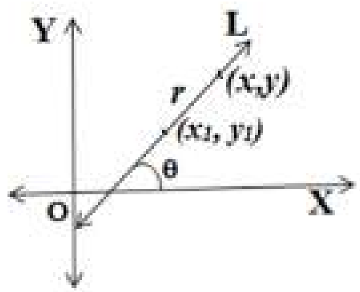
  $$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$



- **Symmetrical (or distance) Form:**

  A straight line passing through the point (x_1, y_1) and inclination θ with x-axis is given by

  $$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

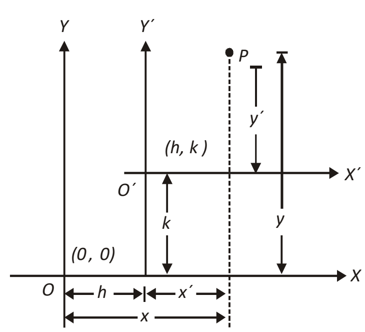


  Where r is the directed distance of any point (x, y) from the point (x_1, y_1).

- **Shifting of Origin:**

  Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

  Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



If the origin is shifted to the point (h, k), then new coordinates (x^1, y^1) and the original coordinates (x, y) of a point are related to each other by the relation

$x' = x - h, y' = y - k$

- Equation of family of lines parallel to $Ax + By + C = 0$ is given by $Ax + By + k = 0$, for different real values of k
- Equation of family of lines perpendicular to $Ax + By + C = 0$ is given by $Bx - Ay + k = 0$, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, for different real values of k.

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.
2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?
3. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6, q). Find p and q.
4. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1)?
5. Show that the points (a, 0), (0, b) and (3a,– 2b) are collinear.
6. Find the equation of straight line cutting off an intercept –1 from *y* axis and being equally inclined to the axes.

7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

8. Find k so that the line 2x + ky – 9 = 0 may be perpendicular to 2x + 3y – 1 = 0

9. Find the acute angle between lines x + y = 0 and y = 0

10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

11. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

12. Fill in the blanks

(a) The equation of a line with slope 1/2 and making an intercept 5 on y-axis is _________.

(b) Equation of line which is parallel to y-axis and at distance 5 units from y-axis is _________.

(c) The length of perpendicular from a point (1, 2) to a line 3x + 4y + 5 = 0 is _________.

(d) The distance between the lines 3x + 4y = 9 and 6x + 8y = 15 is ________.

(e) Angle between lines 5x + y = 7 and –x + 5y = 9 is _______.

(f) Line 5x – 3y = 12 cuts y-axis at _________.

13. True / False

(a) Lines 3x + 2y = 12 and 6x = 4y + 8 are parallel.

(b) Acute angle between two lines with slopes m and m_2 is given by $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

(c) Distance of Point P(–3, 4) from y-axis is 3 units.

(d) Line y = 5 is parallel to y-axis.

(e) x-intercept of line 3x – 4y + 12 = 0 is –4.

(f) $\frac{x}{a} + \frac{y}{b} = 1$ is intercept from of line.

14. The angle between the straight lines $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$ is -

(a) 90° (b) 60°

(c) 75° (d) 30°.

15. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which one of the following is correct?

(a) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(c) $\frac{1}{p} = \frac{1}{a} + \frac{1}{b}$ (d) $\frac{1}{p} = \frac{1}{a} - \frac{1}{b}$.

16. What is the equation of the line passing through (2, –3) and parallel to y-axis?

(a) y = –3 (b) y = 2

(c) x = 2 (d) x = –3.

17. If the lines $3x + 4y + 1 = 0$, $5x + \lambda y + 3 = 0$ and $2x + y - 1 = 0$ are concurrent, then λ is equal to -

    (a) -8 (b) 8

    (c) 4 (d) -4.

18. If $x \cos\theta + y \sin\theta = 2$ is perpendicular to the line $x - y = 3$, then what is one of the value of θ ?

    (a) $\pi / 16$ (b) $\pi / 4$

    (c) $\pi / 2$ (d) $\pi / 3$.

19. The x-intercept and the y-intercept of the line $5x - 7 = 6y$, respectively are -

    (a) $\frac{7}{5}$ and $\frac{7}{6}$ (b) $\frac{7}{5}$ and $\frac{-7}{6}$

    (c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $\frac{-5}{7}$ and $\frac{6}{7}$.

20. If p be the length of the perpendicular from the origin on the straight line $x + 2y = 2p$, then what is the value of b?

    (a) $1/p$ (b) p

    (c) $1/2$ (d) $\sqrt{3}/2$.

21. If we reduce $3x + 3y + 7 = 0$ to the form $x \cos\alpha + y \sin\alpha = 9$, then the value of p is -

    (a) $\frac{7}{2\sqrt{3}}$ (b) $\frac{7}{3}$

    (c) $\frac{3\sqrt{7}}{2}$ (d) $\frac{7}{3\sqrt{2}}$.

22. A straight line through P(1, 2) is such that its intercept between the axes is bisected at P. Its equation is -

(a) $x + y = -1$ (b) $x + y = 3$

(c) $x + 2y = 5$ (d) $2x + y = 4$.

23. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?

(a) 1 (b) 3

(c) 6 (d) 0.

24. The triangle formed by the lines $x + y = 0$, $3x + y = 4$ and $x + 3y = 4$ is -

(a) Isosceles (b) Equilateral

(c) Right angled (d) None of these.

25. What is the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$?

(a) (1, 10) (b) (5, 6)

(c) (6, 5) (d) (7, 4).

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

26. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

27. Determine the equation of line through a point (–4, –3) and parallel to *x*-axis.

28. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices a triangle or not?

29. If a vertex of a triangle is (1, 1) and the midpoints of two sides through this vertex are (–1, 2) and (3, 2). Then find the centroid of the triangle.

30. If the medians through A and B of the triangle with vertices A(0, b), B(0, 0) and C(a, 0) are mutually perpendicular. Then show that $a^2 = 2b^2$.

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

31. If the image of the point (3, 8) in the line px + 3y – 7 = 0 is the point (–1, –4), then find the value of p.

32. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines x + 2y = 5 and x – 3y + 5 = 0

33. The line 2x – 3y = 4 is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

34. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line y = 2x + c. Find c and remaining two vertices.

35. If two sides of a square are along 5x – 12y + 26 = 0 and 5x – 12y – 65 = 0 then find its area.

36. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

37. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

38. What is the value of y so that line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6)?

39. In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line $x + y = 4$?

40. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and –6 respectively.

41. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.

42. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

43. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

44. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

45. The vertices of a triangle are (6, 0), (0,6) and (6,6). Find the distance between its circumcenter and centroid.

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

46. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point (–5, 0) and is at a perpendicular distance of 3 units from origin.

47. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Find the equation of other three sides.

48. If (1, 2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

49. Find the equations of the straight lines which cut off intercepts on x-axis twice that on y-axis and are at a unit distance from origin.

50. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

51. A line is such that its segment between the lines $5x - y + 4 = 0$ and$3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

52. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

53. If the slope of a line passing through to point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

54. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x - y - 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

55. Find points on the line x + y + 3 = 0 that are at a distance of 5 units from the line x + 2y + 2 = 0

56. Show that the locus of the midpoint of the distance between the axes of the variable line x cosα + y sinα = p is $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$, where p is a constant.

57. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ where c is constant. Show that the locus of the foot of perpendicular from the origin to the given line is $x^2 + y^2 = c^2$.

58. A point p is such that the sum of squares of its distance from the axes of coordinates is equal to the square of its distance from the line x – y =1. Find the locus of P.

59. A straight line L is perpendicular to the line 5x – y =1. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L.

60. The vertices of a triangle are [at_1t_2, $a(t_1 + t_3)$], [at_2t_3, $a(t_2 + t_3)$]. Find the orthocentre of the triangle.

61. Two equal sides of an isosceles triangle are given by the equation 7x – y + 3 = 0 and x + y –3 = 0 and its third side pass through the point (1, –10). Determine the equation of the third side.

62. Let A(2, –3) and B(–2, 1) be the vertices of a ΔABC. If the centroid of this triangle moves on the line 2x + 3y = 1. Then find the locus of the vertex C.

63. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2 AC. If the coordinates of D and M are (1, 1) and (2, –1) respectively. Then find the coordinates of A.

64. Find the area enclosed within the curve |x| + |y| = 1.

65. If the area of the triangle formed by a line with coordinates axes is $54\sqrt{3}$ square units and the perpendicular drawn from the origin to the line makes an angle 60° with the x-axis, find the equation of the line.

66. Find the coordinators of the circumcentre of the triangle whose vertices are (5,7), (6,6) and (2, -2).

67. Find the equation of a straight line, which passes through the point (a, 0) and whose $\perp$ distance from the point (2a, 2a) is a.

68. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2} + b^{-2} = p^{-2} + q^{-2}$.

ANSWERS

1. (1, 2)
2. k = 3
3. p = 2, q = –5
4. 3 : 2 (internally)
6. $y = x - 1$ and $y = -x - 1$.
7. $x + y = 7$
8. $\frac{4}{3}$
9. $\frac{\pi}{4}$
10. $\frac{2\pi}{3}$
11. (–3, –1)
12. (a) $y = \frac{x}{2} + 5$
    (b) x = 5
    (c) 16/5
    (d) 3/10
    (e) 90°
    (f) (0, –4)
13. (a) False
    (b) True
    (c) True
    (d) False
    (e) True
    (f) True
14. (a)
15. (a)
16. (c)
17. (b)
18. (b)
19. (b)
20. (d)
21. (d)
22. (d)
23. (c)
24. (a)
25. (b)
26. p = –3, q = –3
27. y + 3 = 0

28. No

29. $\left(1, \frac{7}{3}\right)$

31. 1

32. $\frac{10}{\sqrt{26}}$

33. (1, –5)

34. c = –4, (2, 0), (4, 4)

35. 49 square units

36. $x + y + 5\sqrt{2} = 0, \ x + y - 5\sqrt{2} = 0$

37. 4 square units

38. y = 9

39. 1 : 2

40. 2x – 3y – 6 = 0 and – 3x + 2y – 6 = 0

41. 4 square units

42. (–4, –3)

43. x + 2y = 1

44. x + y – 5 = 0

45. $3\sqrt{2}$

46. 3x – 4y + 15 = 0

47. 4x + 7y – 11 = 0, 7x – 4y + 25 = 0
 7x – 4y – 3 = 0

48. x – 2y + 3 = 0, 2x + y – 14 = 0,
 x – 2y + 13 = 0, 2x + y – 4 = 0
 3x – y – 1 = 0, x + 3y – 17 = 0

49. $x + 2y + \sqrt{5} = 0, \ x + 2y - \sqrt{5} = 0$

50. x = y

51. $107x - 3y - 92 = 0$

52. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$

53. $(-1, -1)$ or $(7, 5)$

54. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$

55. $(1, -4), (-9, 6)$

58. $x^2 + y^2 + 2xy + 2x - 2y - 1 = 0$

59. $x + 5y = \pm 5\sqrt{2}$

60. $\left[-a, a(t_1 + t_2 + t_3 + t_1t_2t_3)\right]$

61. $x - 3y - 31 = 0$, $3x + y + 7 = 0$

62. $\frac{x}{-2} + \frac{y}{1} = 1$

63. $\left(1, \frac{-3}{2}\right)$ or $\left(3, \frac{-1}{2}\right)$

64. $\sqrt{3}$

65. $x + \sqrt{3}y = 18$

66. $(2, 3)$

67. $3x - 4y - 3a = 0$ and $x - a = 0$