

CHAPTER - 15

STATISTICS

KEY CONCEPT

- Range of Ungrouped Data and Discrete Frequency Distribution.
- RANGE = Largest observation – smallest observation.
- Range of Continuous Frequency Distribution.
- RANGE = Upper Limit of Highest Class – Lower Limit of Lowest Class.

- **Mean deviation for ungrouped data or raw data:**

$$M.D. \text{ (about mean)} = \frac{\sum |x_i - \bar{x}|}{n}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D. \text{ (about mean)} = \frac{\sum |x_i - M|}{n}, \text{ where } M \text{ is the Median.}$$

- **Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution):**

$$M.D. \text{ (about mean)} = \frac{\sum |f_i - \bar{x}|}{N}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D. \text{ (about mean)} = \frac{\sum |f_i - M|}{N}, \text{ where } M \text{ is the Median.}$$

Note: $N = \sum f_i$

- Variance is defined as the mean of the squares of the deviations from mean.

- Standard deviation 'σ' is positive square root of variance.

$$\sigma = \sqrt{\text{Variance}}$$

- Variance 'σ²' and standard deviation (SD) σ for ungrouped data

$$\sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2 \Rightarrow \boxed{S.D. = \sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}}$$

- Standard deviation of a discrete frequency distribution

$$S.D. = \sigma = \sqrt{\frac{1}{n} \sum f_i (x_i - \bar{x})^2} = \frac{1}{N} \sqrt{N \sum f_i x_i^2 - (\sum f_i x_i)^2}$$

- Short cut method to find variance and standard deviation

$$\text{Variance} = \sigma^2 = \frac{h^2}{N^2} \left[N \sum f_i y_i^2 - (\sum f_i y_i)^2 \right]$$

$$S.D. = \sigma = \frac{h}{N} \sqrt{N \sum f_i y_i^2 - (\sum f_i y_i)^2}$$

$$\text{Where : } y_i = \frac{x_i - A}{h}$$

- Coefficient Of Variation (C.V.) = c
- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k² times of the original value and standard deviation becomes k times of the original value.
- If each observation is increased by k, where k is positive or negative, then variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.
- The series having higher coefficient of variation is called more variable than the other. While the series having lesser coefficient of variation is called more consistent or more stable. For series with equal means the series with lesser standard deviation is more stable.
- The mean of first 'n' natural number is $\frac{n+1}{2}$.
- The mean of first 'n' even natural numbers = $(n + 1)$

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Fill up the blanks in each of the following:
 - (a) The mean of first ten natural number is _____.
 - (b) The mean of first ten even natural number is _____.
 - (c) The mean of first ten odd natural number is _____.
 - (d) Coefficient of Variation (C.V.) = $\frac{\text{.....}}{\text{Mean}} \times 100, \bar{x} \neq 0$.
 - (e) If the variance of a data is 7225, then the standard deviation of the data is _____.
2. State whether the following are True or False.
 - (a) The range of observations 1, 2, 5, 3, 0, 8, 10, 9 is eight (8).
 - (b) The mean deviation about Mean for 1, 3, 5, 7, 9 is 2.4
 - (c) The mean deviation about Median for 1, 3, 5, 7, 9 is 5.

- (d) If the mean of a, b, c, d, e is 10, then mean of $(a + 3), (b + 3), (c + 3), (d + 3), (e + 3)$ is also 10.
- (e) If the Variance of a, b, c, d, e is 10, then variance of $(a + 3), (b + 3), (c + 3), (d + 3), (e + 3)$ is also 10.
3. The sum of the squares of deviation for 10 observations taken from their mean 50 is 250. Find Standard Deviation.
4. The sum of the squares of deviation for 10 observations taken from their mean 25 is 500. Find Variance.
5. If the variance of 14, 18, 22, 26, 30 is 'k', then find the variance of 28, 36, 44, 52, 60.

Note: Q.6 – Q.10 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

6. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new mean -
(a) 12 (b) 16
(c) 24 (d) 48.
7. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new standard deviation -
(a) 4 (b) 8
(c) 16 (d) 32.
8. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new mean-
(a) 25 (b) 29
(c) 30 (d) 35.

9. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new variance -
- (a) 4 (b) 14
(c) 16 (d) 25.

10. Match the following:
If the mean of x_1, x_2, \dots, x_{20} is 10.

	Column-1		Column-2
A	mean of $2x_1, 2x_2, \dots, 2x_{20}$	P	0
B	mean of $(-3x_1 + 32), (-3x_2 + 32), \dots, (3x_{20} + 32)$	Q	2
C	mean of $(x_1 + 2), (x_2 + 2), \dots, (x_{20} + 2)$	R	12
D	mean of $(x_1 - 10), (x_2 - 10), \dots, (x_{20} - 10)$	S	20

- (a) $A \rightarrow P, B \rightarrow Q, C \rightarrow R, D \rightarrow S$
 (b) $A \rightarrow S, B \rightarrow Q, C \rightarrow R, D \rightarrow P$
 (c) $A \rightarrow Q, B \rightarrow S, C \rightarrow R, D \rightarrow P$
 (d) $A \rightarrow S, B \rightarrow Q, C \rightarrow P, D \rightarrow R$

Section - B

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

11. Find the Variance of First 10 Natural Numbers.
12. Find the Variance of First 5 Multiples of 6.
13. Find the Standard Deviations of First 10 Even Natural numbers.
14. Find the Standard deviation for the following data:
10, 20, 30, 40, 50, 50, 60, 70, 80, 90

15. Find the variance for the following Data:

Class-Interval	Frequency
0 - 10	1
10 - 20	2
20 - 30	3
30 - 40	3
40 - 50	1

Section - C

LONG ANSWER TYPE – I QUESTIONS (4 MARKS)

16. In a series of '2p' observations, half of the observations are equal 'a' each and remaining half equal (-a) each. If the standard deviation of the observations is 2, then find the value of |a|.
17. In the following Distribution

x	f
A	2
2A	1
3A	1
4A	1
5A	1
6A	1

Where A is positive integer, has a variance of 160. Determine the value of A.

18. Find the mean deviation from mean of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

19. Find the Variance and Standard Deviation of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.
20. Consider the first 10 positive integers. If we multiply each number by -1 and then add 1 to each number, find the variance of the numbers so obtained.
21. Coefficients of variation of two distributions A and B are 60 and 80 respectively while their standard deviations are 21 and 36 respectively. What are their means?
22. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6. Find the other two observations.
23. Calculate the possible values of 'x' if standard deviation of the numbers 2, 3, $2x$ and 11 is 3.5.
24. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.
25. Suppose a population A has 100 observations 101, 102,, 200. Another population B has 100 observations 151, 152,, 250. If V_A and V_B represent the variances of the two populations respectively then find the ratio of V_A and V_B .

Section - D

LONG ANSWER TYPE – II QUESTIONS (6 MARKS)

26. Calculate the mean deviation about mean for the following data.

X	2	4	6	8	10	12	14	16
f	2	2	4	5	3	2	1	1

27. If for a distribution $\sum(x-5) = 3$, $\sum(x-5)^2 = 43$ and the total number of item is 18, find the mean and standard deviation.

28. Calculate the mean deviation about median for the following data:

X	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	4

29. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test :

X	0	1	2	3	4	5
f	$p - 2$	p	p^2	$(p + 1)^2$	$2p$	$2p + 1$

where p is positive integer. Determine the mean and standard deviation of the marks.

30. Calculate the mean deviation about mean

Class Interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
f	2	3	8	14	8	3	2

31. Mean and standard deviation of 100 observations were found to be 40 and 10 respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively. Find correct standard deviation.

32. Calculate the mean deviation about mean for the following data:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	8	15	16	6

33. Calculate the mean deviation about median for the following data

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
f	8	10	10	16	14	2

34. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results:

Number of observations = 25,

mean = 18.2 seconds

Standard deviation = 3.25 seconds

Further another set of 15 observations x_1, x_2, \dots, x_{15} ,

also in $\sum_{i=1}^{15} x_i^2 = 5524$.

Calculate the standard deviation based on all 40 observations.

35. Find the coefficient of variation of the following data:

Class Interval	f
20 - 29	5
30 - 39	12
40 - 49	15
50 - 59	20
60 - 69	18
70 - 79	10
80 - 89	6
90 - 99	4

ANSWERS

1. (a) 5.5
(b) 11
(c) 10
(d) Standard Deviation
(e) 85
2. (a) False
(b) True
(c) False
(d) False
(e) True
3. 5
4. 25
5. 4 k
6. (d)
7. (c)
8. (d)
9. (c)
10. (b)
11. 8.33
12. 72
13. $\sqrt{33}$
14. $10\sqrt{6}$
15. $\sqrt{129}$
16. 2
17. $A = 7$
18. $\frac{(n-1)(d-1)}{2}$
19. Variance = $\frac{(n^2-1)}{12}d^2$
Standard Deviation = $d\sqrt{\frac{(n^2-1)}{12}}$
20. 8.25
21. 35, 45
22. 4, 9
23. $3, \frac{7}{3}$
24. 6.5, 2.5
25. 1 : 1

26. 2.8
27. Mean = 5.17,
Standard Deviation = 1.53
28. 10.1
29. Mean = 2.8,
Standard deviation = 1.12
30. 10
31. 10.24
32. 9.44
33. 11.44
34. 3.87
35. 31.24