CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- A sequence is said to be a progression if the term of the sequence can be expressed by some formula
- **Arithmetic Progression:** A sequence is called an arithmetic progression if the difference of a term and previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=d) for all $n \in N$.
- General A.P. is a, a + d and a + 2d, ...
- $a_n = a + (n - 1)d = n^{th}$ term of A.P. $= l$
- S_n = Sum of first n terms of A.P. $= \frac{n}{2}[a + l]$, where l = last term N.

$$= \frac{n}{2}[2a + (n-1)d]$$

- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P. = ak, bk, ck also in A.P., k ≠ 0, $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where k ≠ 0.
- Arithmetic mean between a and b is $\frac{a+b}{2}$.
- If $A_1, A_2, A_3, ... A_n$ are n numbers inserted between a and b, such that the resulting sequence is A.P.

then, $A_n = a + n \cdot \frac{b-a}{n+1}$

- $S_k - S_{k-1} = a_k$
- $a_m = n, a_n = m \Rightarrow a_r = m + n - r$
- $S_m = S_n \Rightarrow S_{m+n} = 0$
- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$
- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term.
- If a, b, c are in A.P. then 2b = a + c.
- If four terms of A.P. are to be taken then we choose then as a – 3d, a – d, a + d, a + 3d.
- If five terms of A.P are to be taken, then we choose then as a – 2d, a – d, a, a + d, a + 2d.
- **G.P. (Geometrical Progression)**

  (i) a, ar, ar^2, ...(General G.P.)

  (ii) $a = ar^{n-1}$

  (iii) $S_n = \frac{a(r^n - 1)}{r - 1}$, $r \neq 1$

- If a, b, c are in G.P., then $b^2 = ac$.
- Geometric mean between a and b is $\sqrt{ab}$.
- Reciprocals of terms in GP always form a G.P.

- If G_1, G_2, G_3, ...G_n are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}} \quad 1 \le k \le n$$

- If three terms of G.P. are to be taken, then we those then as $\frac{a}{r}$, a, ar.

- If four terms of G.P. are to be taken, then we choose then as $\frac{a}{r^3}, \frac{a}{r}$, a, ar.

- If a, b, c are in G.P. then ak, bk, ck are also in G.P., where $k \neq 0$ and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ also in G.P. where $k \neq 0$.

- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.

- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.

- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1-r}$.

- **Special Series:**

  (i) $$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

  (ii) $$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

- Let $a_1, a_2, a_3, \ldots\ldots\ldots$ be a sequence, then the expression $a_1 + a_2 + a_3 + \ldots\ldots\ldots$ is called series associated with given sequence?

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If n^{th} term of an A.P. is $6n - 7$ then write its 50^{th} term.

2. If $S_n = 3n^2 + 2n$, then write a_2

3. Which term of the sequence 3, 10, 17, ... is 136?

4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.

6. Which term of the G.P. $2, 1, \frac{1}{2}, \frac{1}{4} \ldots\ldots\ldots\ldots$ is $\frac{1}{1024}$?

7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.

8. In G.P. $2\sqrt{2}, 4, \ldots\ldots\ldots, 128\sqrt{2}$, find the 4^{th} term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term.

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}, \ldots\ldots\ldots$

11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ .... upto infinity.

12. Write the value of $0.\overline{3}$

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. Fill in the blanks

(a) If 7^{th} and 13^{th} terms of an A.P. be 34 and 64 respectively, then 18^{th} term is _______.

(b) Geometric mean of 4 and 9 is ________.

(c) If the sum of p terms of an A.P. is q and sum of q terms is p, then the sum of p + q terms will be ________.

(d) Sum of infinity of sequence $5, \frac{5}{3}, \frac{5}{9}, \ldots\ldots\ldots$ _______.

(e) If a, b, c are in A.P. and x, y, z are in G.P., then the value of $x^{b-c} \times y^{c-a} \times z^{c-a}$ is ________.

(f) The two geometric means between numbers 1 and 64 are ____________.

15. True / False

(a) Common difference of an A.P. is always positive.

(b) n^{th} term of a G.P. is a + (n – 1)d.

(c) $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^n = \frac{n(n-1)(2n-1)}{6}$.

(d) 2 + 4 + 6 + ………… + 2n = n(n + 1).

(e) 0.9, 0.99, 0.999, …………. from G.P.

(f) In a G.P. 5_∞ is always not defined.

16. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5, then the number of side is -

(a) 8 (b) 10

(c) 9 (d) 6.

17. α and β are the roots of the equation $x^2 - 3x + a = 0$ and γ and δ are the roots of the equation $x^2 - 12x + b = 0$. If α, β, γ and δ form an increasing G.P., then (a, b) -

(a) (3, 12) (b) (12, 3)

(c) (2, 32) (d) (4, 16).

18. If A be the arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then -

(a) S = nA (b) A = nS

(c) A = S (d) None of these.

19. In an A.P., the m^{th} term is 1/n and n^{th} term is 1/m. What is its $(mn)^{th}$ term?

(a) 1/(mn) (b) m/n

(c) n/m (d) 1.

20. If n geometric means be inserted between a and b, then the n^{th} geometric mean will be -

(a) $a\left[\frac{b}{a}\right]^{\frac{n}{n-1}}$ (b) $a\left[\frac{b}{a}\right]^{\frac{n-1}{n}}$

(c) $a\left[\frac{b}{a}\right]^{\frac{n}{n+1}}$ (d) $a\left[\frac{b}{a}\right]^{\frac{1}{n}}$.

21. What is the 15^{th} term of the series 3, 7, 13, 21, 31, 43 -

(a) 205 (b) 225

(c) 238 (d) 241.

22. If a, b and c are in G.P., then $\frac{1}{a^2 - b^2} + \frac{1}{b^2}$ is -

(a) $\frac{1}{c^2 - b^2}$ (b) $\frac{1}{b^2 - c^2}$

(c) $\frac{1}{c^2 - a^2}$ (d) $\frac{1}{b^2 - a^2}$.

23. If the 10^{th} term of a G.P. is 9^{th} and 4^{th} term is 4, then what is its 7^{th} term -

(a) 6 (b) 14

(c) 27/14 (d) 56/15.

24. If the arithmetic and geometric means of two numbers are 10 and 8 respectively, then one number exceeds the other number by -

(a) 8 (b) 10

(c) 12 (d) 16.

25. What is the sum of numbers lying between 107 and 253, which are divisible by 5 -

(a) 5220 (b) 5210

(c) 5200 (d) 5000.

26. Sum of all two digit numbers which when divided by 4 yield unity as remainder is -

(a) 1200 (b) 1210

(c) 1250 (d) None of these.

27. The first and last terms of A.P. are 1 and 11. If the sum of its term is 36, then the number of terms will be -

(a) 5 (b) 6

(c) 7 (d) 8.

28. If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are -

(a) 5, 10, 15, 20 (b) 4, 10, 16, 22

(c) 3, 7, 11, 15 (d) None of these.

29. If the first, second and last term of an A.P. are a, b and 2a respectively, then its sum is -

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) None of these.

30. If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of this G.P. is -

(a) $\frac{p-q}{q-r}$ (b) $\frac{q-r}{p-q}$

(c) pqr (d) None of these.

31. The n^{th} term of a G.P. is 128 and the sum of its n term is 225. If its common ratio is 2, then the first term is -

(a) 1 (b) 3

(c) 8 (d) None of these.

32. If A be one A.M. and p, q be two GM's between two numbers, then 2A is equal to -

(a) $\frac{p^3+q^3}{pq}$ (b) $\frac{p^3-q^3}{pq}$

(c) $\frac{p^2+q^2}{2}$ (d) $\frac{pq}{2}$.

33. In a G.P. if the $(m + n)^{th}$ term is p and $(m - n)^{th}$ term is q, then its m^{th} term is -

(a) O (b) pq

(c) $\sqrt{pq}$ (d) $\frac{1}{2}(p+q)$.

34. If $\sum n = 210$, then $\sum n^2 =$

(a) 2870 (b) 2160

(c) 2970 (d) None of these.

35. The sum of 10 terms of the series $\sqrt{2}+\sqrt{6}+\sqrt{18}+......$ is -

(a) $121\left(\sqrt{6}+\sqrt{2}\right)$ (b) $243\left(\sqrt{3}+1\right)$

(c) $\frac{1}{\sqrt{3}-1}$ (d) $243\left(\sqrt{3}-1\right)$.

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

36. Write the n^{th} term of the series, $\frac{3}{7.11^2}+\frac{5}{8.12^2}+\frac{7}{9.13^2}+.........$

37. Find the number of terms in the A.P. 7, 10, 13, ..., 31.

38. In an A.P.,

8, 11, 14, ... find $S_n - S_{n-1}$

39. Find the number of squares that can be formed on chess board?

40. Find the sum of given terms:-

    (a) 81 + 82 + 83 .... + 89 + 90

    (b) 251 + 252 + 253 + ... + 259 + 260

41. (a) If a, b, c are in A.P. then show that 2b = a + c.

    (b) If a, b, c are in G.P. then show that $b^2 = a \cdot c$.

42. If a, b, c are in G.P. then show that $a^2 + b^2$, ab + bc, $b^2 + c^2$ are also in G.P.

Section - C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

43. Find the least value of n for which

    $1 + 3 + 3^2 + ... + 3^{n-1} > 1000$

44. Find the sum of the series

    $(1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + \ldots\ldots\ldots$

45. Write the first negative term of the sequence 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ...

46. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11^{th} term from the end.

47. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

48. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

49. Find the sum of the sequence,

    72 + 70 + 68 + ………. + 40

50. If in an A.P $\frac{a_7}{a_{10}} = \frac{5}{7}$, find $\frac{a_4}{a_7}$.

51. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

52. Solve : 1 + 6 + 11 + 16 + ... + x = 148

53. The ratio of the sum of n terms of two A.P.'s is (7n – 1) : (3n + 11), find the ratio of their 10^{th} terms.

54. If the I^{st}, 2^{nd} and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

55. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{a}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term] abc.

56. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

57. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

58. Find the sum to infinity of the series :

    $1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \ldots\ldots \infty$

59. If A = 1 + r^a + r^{2a} + ... up to infinity, then express r in terms of 'a' and 'A'.

60. Find the sum of first terms of the series 0.7 + 0.77 + 0.777 + …..

61. If $x = a + \frac{a}{r} + \frac{a}{r^2} + ... + \infty$; $y = b - \frac{b}{r} + \frac{b}{r^2} - .... \infty$ and

$z = c + \frac{c}{r^2} + \frac{c}{r^4} + ...\infty$ Prove that $\frac{xy}{z} = \frac{ab}{c}$.

62. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

63. Prove that $0.003\overline{1} = \frac{7}{225}$.

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

64. If a, b, c are in G.P. that the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

65. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b + c, c + a, a + b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

66. If the numbers a^2, b^2 and c^2 are given to be in A.P., show that $\frac{1}{b+c}$, $\frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

67. Show that : $0.3\overline{56} = \frac{353}{990}$

68. Find the sum of n terms of series :

$3 + 5 + 9 + 15 + 23 + \ldots\ldots\ldots\ldots$ n terms

69. Find the sum of n terms of series :

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 \ldots\ldots\ldots\ldots$ n terms

70. The fourth term of a G.P. is 4. Find product of its first seven terms.

71. If A_1, A_2, A_3, A_4 are four A.M's between $\frac{1}{2}$ and 3, then prove $A_1 + A_2 + A_3 + A_4 = 7$.

72. If S_n denotes the sum of first n terms of an A.P. If $S_{2n} = 5S_n$, then prove $\frac{S_{6n}}{S_{3n}} = \frac{17}{4}$.

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

73. Prove that the sum of n numbers between a and b such that the resulting series becomes A.P. is $\frac{n(a+b)}{2}$.

74. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of

the first square is 15 cm, then find the sum of the areas of all the squares so formed.

75. If a, b, c are in G.P., then prove that $\frac{1}{a^2 - b^2} - \frac{1}{b^2 - c^2} = -\frac{1}{b^2}$.

[**Hint :** Put b = ar, c = ar^2]

76. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

77. If a is A.M. of b and c and c, G_1, G_2, b are in G.P., then prove that $G_1^3 + G_2^3 = 2abc$

78. Find the sum of the series,

1.3.4 + 5.7.8 + 9.11.12 + ... upto n terms.

79. Evaluate: $\sum_{r=1}^{10} (2r-1)^2$

80. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

81. If $10^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \ldots\ldots + 10(11)^9 = k.(10)^9$, then find the value of k such that $k \in N$.

82. Find the sum of first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots..n$ terms.

83. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

84. Show that if the positive number a, b, c are in A.P. so are the numbers $\frac{1}{\sqrt{a}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

85. Find the sum of the series: $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\frac{9}{16}.........\infty$

86. If the sum of first 'n' terms of an A.P. is $c \cdot n^2$ then prove that the sum of squares of these 'n' terms is $\frac{nc^2\left(4n^2-1\right)}{3}$.

87. Let 'p' and 'q' be the roots of the equation $x^2 - 2x + A = 0$ and let 'r' and 's' be the roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ are in A.P., then prove that $A = -3$ and $B = 77$.

88. If S_1, S_2, S_3 …… S_n are the sums of infinite geometric series whose first terms are 1, 2, 3, …… n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ ...... $\frac{1}{n+1}$ respectively, then show that the value of $S_1^2+S_2^2+S_3^2+.......S_{2n-1}^2=\frac{1}{6}(2n)(2n+1)(4n+1)-1.$

89. If p^{th}, q^{th} and r^{th} terms of a G.P. are equal and are x, y and z respectively, then prove that $x^{y-z} \cdot y^{z-x} \cdot z^{x-y} = 1$.

90. The sum of infinite G.P. is 57 and sum of their cubes is 9747, find the G.P.

91. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 9.

ANSWERS

1. 293
2. 11
3. 20th
4. 0
5. 4n + 5
6. 12th
7. $\frac{9}{2}$
8. 64
9. 3
10. $20\left(1-\frac{1}{2^8}\right)$
11. 5
12. $\frac{1}{3}$
13. $\frac{2}{3}$
14. (a) 89
    (b) 6
    (c) –(p + q)
    (d) $\frac{15}{2}$
    (e) 1
    (f) 4 and 16
15. (a) False
    (b) False
    (c) True
    (d) True
    (e) False
    (f) False
16. (c)
17. (c)
18. (a)
19. (d)
20. (c)
21. (d)
22. (b)
23. (a)

24.	(c)	25.	(a)
26.	(b)	27.	(b)
28.	(a)	29.	(c)
30.	(b)	31.	(a)
32.	(a)	33.	(c)
34.	(a)	35.	(a)
36.	$\frac{2n+1}{(n+6)(n+10)^2}$	37.	9
38.	3n + 5	39.	204
40.	(a) 855 (b) 2555	43.	n = 7
44.	$\frac{n}{1-x}-\frac{x^2(1-x^n)}{(1-x)^2}$	45.	$-\frac{1}{4}$
46.	102, 367	47.	33
48.	7999	49.	952
50.	$\frac{3}{5}$	51.	11
52.	36	53.	33 : 17
54.	$\frac{(b+c-2a)(a+c)}{2(b-a)}$		
56.	5, 10, 20, ...; or 20, 10, 5,	57.	18, 6, 2; or 2, 6, 18
58.	6	59.	$\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$

60. $\frac{7}{81}\left[9n-1+10^{-n}\right]$

62. $\frac{15}{7}\left(2^{n}-1\right)$

68. $\frac{n(n^2+8)}{3}$

69. $\frac{n(n+1)}{2}$

70. 16384

74. 450 cm^2

76. 16, 4

78. $\frac{n(n+1)}{3}(48n^2-16n-14)$

79. 1330

80. $19, \frac{38}{3}, \frac{76}{9}, \ldots\ldots$

81. k = 100

82. $n + 2^{-n} - 1$

83. $r = 2+\sqrt{3}$

85. $\frac{2}{9}$

90. 19, 38/3, 76/9, ……

91. 1, 3, 9