CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- A meaningful sentence which can be judged to be either true or false is called a statement.
- A statement involving mathematical relations is called as mathematical statement.
- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Induction being with observations. From observations we arrive at tentative conclusions called conjectures. The processes of induction help in proving the conjectures which may be true.
- Statements like

(i) $1+2+3+.........+n=\frac{n(n+1)}{2} \;\; \forall \; n \in N.$

(ii) $2^n \leq 2 \;\; \forall \; n \in N.$

(iii) If n(A) = n then number of all subsets of A = $2^n \;\forall\; n \in N$.

(iv) $Sn = \frac{a(r^n - 1)}{r - 1}$ where Sn is sum of n terms of G.P, a = 1st term and r = common ratio. Are all concerned with $n \in N$ which takes values 1, 2, 3, …….. Such statements are denoted by P(n). By giving particular values to 'n', we get particular statement as P(1), P(2), …….. P(k) for some $k \in N$.

Principle of mathematical Induction:

Let P(n) be any statement involving natural number n such that

(i) P(1) is true, and

(ii) If P(k) is true $\Rightarrow$ P(k + 1) is true for some $k \in N$. that is P(K + 1) is true whenever P(K) is true for some $k \in N$ then P(n) is true $\forall\, n \in N$.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARKS)

1. Let P(n): $n^2 + n$ is even. Is P(1) true?

2. Let P(n): n(n+1) (n+2) is divisible by 3. What is P(3)?

3. Let P(n): $n^2 > 9$. Is P(2) true?

4. If $10^n + 3.4^{n+2} + K$ is divisible by 9 for all $n \in N$, then the least positive integral value of K is –

   (a) 5 (b) 3 (c) 7 (d) 1

5. For all $n \in N$, $3.5^{2n+1} + 3^{2n+1}$ is divisible by –

   (a) 19 (b) 17 (c) 23 (d) 25

6. If $x^n - 1$ is divisible by x – k, then least positive integral value of K is –

   (a) 1 (b) 2 (c) 3 (d) 4

7. State the following statement is true or false –

   "Let P(n) be a statement and let $P(k) \Rightarrow P(k + 1)$, for some natural K, then P(n) is true for all $n \in N$".

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

8. Give an example of a statement such that P(3) is true but P(4) is not true.

9. If P(n) : $1 + 4 + 7 + \ldots\ldots + (3n - 2) = \frac{1}{2}n(3n - 1)$. Verify P(n) for n = 1, 2.

10. If P(n) is the statement "$n^2 - n + 41$ is Prime" Prove that P(1) and P(2) are the but P(41) is not true.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Prove the following by using the principle of mathematical induction $\forall\, n \in N$. (For Q.11 – Q.32)

Type-1

11. $3.6 + 6.9 + 9.12 + \ldots\ldots + 3n(3n + 3) = 3n(n + 1)(n + 2)$

12. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\ldots\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$

13. $a + (a + d) + (a + 2d) + \ldots\ldots + [a + (n-1)d] = \frac{n}{2}[2a+(n-1)d]$

HOTS

14. $7 + 77 + 777 + \ldots + \text{to } n \text{ terms} = \frac{7}{81}(10^{n+1} - 9n - 10)$

15. $\sin x + \sin 2x + \sin 3x + \ldots\ldots\ldots \sin nx = \dfrac{\sin\left(\dfrac{n+1}{2}x\right)\sin\dfrac{nx}{2}}{\sin\dfrac{x}{2}}$.

16. $\sin x + \sin 3x + \ldots\ldots\ldots \sin(2n-1)x = \dfrac{\sin^2 nx}{\sin x}$.

17. $\cos\alpha.\cos 2\alpha.\cos 4\alpha \ldots\ldots\ldots\ldots \cos\left(2^{n-1}\alpha\right) = \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}$.

18. $1^2 + 2^2 + 3^2 \ldots\ldots\ldots n^2 \dfrac{n(n+1)(2n+1)}{6}$

Type II

19. $2^{3n-1} - 1$ is divisible by 7.

20. 3^{2n} when divided by 8 leaves the remainder 1.

21. $4^n + 15n - 1$ is divisible by 9.

HOTS

22. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9

23. $11^{n+2} + 12^{2n+1}$ is divisible by 133

24. $x^n - y^n$ is divisible by (x–y) if x and y are any two distinct integers.

25. Given that $5^n - 5$ is divisible by 4 $\forall$n $\in$N. Prove that $2.7^n + 3.5^n - 5$ is a multiple of 24.

26. $7^{2n} + 2^{3n-3}.3^{n-1}$ is divisible by 25.

Type III

27. $2^{n+1} > 2n+1$
28. $3^n > 2^n$
29. $n < 2^n$

HOTS

30. $1+\frac{1}{4}+\frac{1}{9}+.......+\frac{1}{n^2} < 2-\frac{1}{n}$.
31. $(1+x)^n \geq 1+nx \ \ where \ x > -1$.
32. $2^{n+3} \leq (n+3)!$

ANSWER

1. True
2. P(3) : 3 (3 + 1) (3 + 2) is divisible by 3
3. NO.
4. (a)
5. (b)
6. (a)
7. True
8. P(n) : $3n^2$ + n is divisible by 3 and soon
9. P(1) and P(2) are true.