

## CHAPTER - 6

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# LINEAR INEQUALITIES

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### KEY POINTS

- ▶ **Inequalities:** A statement involving ' $<$ ', ' $>$ ', ' $\geq$ ' or ' $\leq$ ' is called inequality. Eg.,  $7 > 5$ ,  $5x - 3 \leq 4$ 
  - Inequalities which do not involve variables are called numerical inequalities.  
Eg.,  $5 > 9$  and  $13 > -2$
  - Inequalities which involve variables are called literal inequalities.  
Eg.,  $3x - 4 \leq 15$  and  $4x - 3y \geq 5$
  - Inequalities involving the symbols ' $>$ ' or ' $<$ ' are called strict inequalities.
  - Inequalities involving the symbols ' $\geq$ ' or ' $\leq$ ' are called slack inequalities.
- ▶ **Linear inequalities in one variable:** The inequalities of form  $ax + b + 0$ ,  $ax + b < 0$ ,  $ax + b \geq 0$  or  $ax + b \leq 0$ ;  $a \neq 0$  are called linear inequalities in one variable.  
Eg.,  $4x - 5 \geq 20$  and  $-3x - 2 < 5x + 4$
- ▶ **Algebraic solutions of linear inequalities in one variables:**
  - **Rule-1**  
Equal numbers may be added (or subtracted from) to both sides without affecting sign of inequalities.


● **Rule-2**


- (i) If both sides of inequality are multiplied (or divided) by same positive number, then sign of inequality remains unchanged.
- (ii) If both sides are multiplied (or divided) by any negative number, then sign of inequality is reversed.


Eg., (i)  $4x \geq 8 \Rightarrow \frac{4x}{4} \geq \frac{8}{4} \Rightarrow x \geq 2$


(ii)  $-4x \geq 8 \Rightarrow \frac{-4x}{-4} \geq \frac{8}{-4} \Rightarrow x \geq 2$


► **Graphical representation of solutions on number line:**


(i)  $x > a \Leftrightarrow a < x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$  

(ii)  $x < a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$  

(iii)  $x \geq a \Leftrightarrow a \leq x < \infty \Leftrightarrow x \in [a, \infty) \Leftrightarrow$  

(iv)  $x \leq a \Leftrightarrow -\infty < x \leq a \Leftrightarrow x \in (-\infty, a] \Leftrightarrow$  

(v)  $a < x < b \Leftrightarrow x \in (a, b) \Leftrightarrow$  

(vi)  $a \leq x \leq b \Leftrightarrow x \in [a, b] \Leftrightarrow$  

- **Linear inequalities in two variables:** The inequalities of form  $ax + by + c > 0$ ,  $ax + by + c < 0$ ,  $ax + by + c \geq 0$  or  $ax + by + c \leq 0$  are linear inequalities in two variables. ( $a, b \neq 0$ )

Eg.,  $4x - 3y < 15$  and  $-4x + 15y + 3 \geq 4$

► **Graphical solution of linear inequalities in two variables**

- A line divides the Cartesian plane into two parts. Each part is known as a half plane.
- The region containing all the solutions of the inequality is called solution region.
- In order to identify the half plane represented by an inequality (solution region), it is just sufficient to take any point (a, b) not on the line and check whether it satisfy the inequality or not.
- If it satisfies, then the regions containing that point (a, b) is solution region.
- If it does not satisfy, then the other region is solution region.
- If inequality contains ' $\geq$ ' or ' $\leq$ ', then points on line  $ax + by = c$  are also included in solution region. In this case we draw dark line while sketching graph of  $ax + by = c$ .
- If inequality contains ' $>$ ' or ' $<$ ', then points on line  $ax + by = c$  are not included in solution region. In this case we draw dotted line while sketching graph of  $ax + by = c$ .

**Note:** While solving system of linear inequalities in two variables, the common of solution regions of each inequality is solution region of system.

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Solve  $5x < 24$  when  $x \in \mathbb{N}$
2. Solve  $3 - 2x < 9$  when  $x \in \mathbb{R}$ . Express the solution in the form of interval.
3. Show the graph of the solution of  $2x - 3 > x - 5$  on number line.

4. Solve  $\frac{1}{x-2} \leq 0$ ,  $x \in \mathbb{R}$ .
5. Solve  $0 < \frac{-x}{3} < 1$ ,  $x \in \mathbb{R}$
6. Solve  $-3 \leq -3x + 2 < 4$ ,  $x \in \mathbb{R}$ .
7. Draw the graph of the solution set of  $x + y \geq 4$ .
8. Draw the graph of the solution set of  $x < y$ .
9. Fill in the blanks
- (a) If  $3x + 17 \leq 2(1 - x)$ , then  $x \in$  \_\_\_\_\_.
- (b) If  $\frac{x^2}{x-2} > 0$ , then  $x \in$  \_\_\_\_\_.
- (c) If  $x^2 \leq 4$ , then  $x \in$  \_\_\_\_\_.
- (d) Statement  $4x - 3 \geq 10$  is \_\_\_\_\_.
- (e) If  $|x| > 5$ , then  $x \in$  \_\_\_\_\_.
10. If  $-4x > 20$  and  $x \in \mathbb{Z}^+$  then  $x$  belongs to -
- (a)  $\{-6, -7, -8, \dots\}$  (b)  $\phi$
- (c)  $\{-4, -3, -2, -1\}$  (d)  $\{1, 2, 3, 4, \dots\}$ .
11. If  $\frac{x-3}{x-2} > 0$  then  $x$  belongs to -
- (a)  $(-\infty, 3) \cup (5, \infty)$  (b)  $(-\infty, -3) \cup (-5, \infty)$
- (c)  $(-\infty, 3] \cup [5, \infty)$  (d)  $(3, 5)$

12. Solution set for inequality  $|x - 1| \leq 5$  is -  
(a)  $[-6, 4]$  (b)  $[-4, 0]$   
(c)  $[-4, 6]$  (d)  $[0, 6]$ .
13. Solution set for inequality  $\frac{1}{x-2} < 0$  is -  
(a)  $(2, \infty)$  (b)  $\phi$   
(c)  $(0, 2)$  (d)  $(-\infty, 2)$ .
14. Solution set for inequality  $5x - 3 < 3x + 1$ ,  $x \in \mathbb{N}$  is -  
(a)  $(-\infty, 2)$  (b)  $\{0, 1, 2\}$   
(c)  $\{1\}$  (d)  $\phi$ .
15. Which of the following point lies in solution region of inequality  $3x - y \leq 5$  ?  
(a)  $(5, 1)$  (b)  $(1, 5)$   
(c)  $(2, 0)$  (d)  $(2, -1)$ .
16. If  $x > 0$  and  $y < 0$  then  $(x, y)$  lies in -  
(a) I quadrant (b) II quadrant  
(c) III quadrant (d) IV quadrant.
17. If  $x^2 > 9$  then  $x$  belongs to -  
(a)  $(-3, 3)$  (b)  $(0, 3)$   
(c)  $(3, \infty)$  (d)  $(-\infty, -3) \cup (3, \infty)$ .
18. Solution set for inequality  $-8x \leq 5x - 3 < 7$  is -  
(a)  $(-1, 2)$  (b)  $(2, 3)$   
(c)  $[-1, 2)$  (d)  $[2, 3]$ .

19. True / False

- (a) Solution set for inequality  $2x - 6 \leq 0$  is  $(0, 3]$ .
- (b) Solution set for inequality  $-8 \leq 5x - 3 < 7$  is  $[-1, 2)$ .
- (c) Inequality  $4x - 7 \geq 3x + 4$  is slack inequality.
- (d) Inequality  $4x - 7 < 8$  is numerical inequality.

**SHORT ANSWER TYPE QUESTIONS (2 MARKS)**

20. Solve  $\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0, \quad x \in \mathbb{R}.$

21. Solve  $\frac{x+3}{x-1} > 0, \quad x \in \mathbb{R}.$

**Solve the inequalities for real  $x$  and represent solution on number line**

22.  $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}, \quad x \in \mathbb{R}.$

23.  $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2, \quad x \in \mathbb{R}.$

24.  $-5 \leq \frac{2-3x}{4} \leq 9, \quad x \in \mathbb{R}.$

25.  $\frac{x+3}{x-2} > 0, \quad x \in \mathbb{R}$

26.  $\frac{x-3}{x-5} > 2$

27.  $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$

28.  $\frac{2x+3}{x-3} \leq 4$
29. Find the pair of consecutive even positive integers which are greater than 5 and are such that their sum is less than 20.

**SHORT ANSWER TYPE QUESTIONS (4 MARKS)**

30. A company manufactures cassettes and its cost and revenue functions are  $C(x) = 26000 + 30x$  and  $R(x) = 43x$  respectively, where  $x$  is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realise some profit.
31. While drilling a hole in the earth, it was found that the temperature ( $T^{\circ}\text{C}$ ) at  $x$  km below the surface of the earth was given by  $T = 30 + 25(x - 3)$ , when  $3 \leq x \leq 15$ .  
Between which depths will the temperature be between  $200^{\circ}\text{C}$  and  $300^{\circ}\text{C}$ ?
32. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3<sup>rd</sup> reading that will result in acidity level being normal.

**Solve the following systems of inequalities for all  $x \in \mathbb{R}$**

33.  $2(2x+3) - 10 < 6(x-2)$ ,  $\frac{2x-3}{4} + 6 \geq 4 + \frac{4x}{3}$
34.  $|2x-3| \leq 11$ ,  $|x-2| \geq 3$
35.  $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ ,  $\frac{7x-1}{3} - \frac{7x+2}{6} > x$

36. Solve  $\frac{|x|-1}{|x|-2} \geq 0$   $x \in \mathbb{R}$ ,  $x \neq \pm 2$
37. Solve for real  $x$ ,  $|x+1|+|x| > 3$
38. In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks he should score in fifth paper.
39. A milkman has 80% milk in this stock of 800 litres of adulterated milk. How much 100% pure milk is to be added to it so that purity is between 90% and 95%?
40.  $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ ,  $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$
41.  $\frac{x}{2x+1} \geq \frac{1}{4}$ ,  $\frac{6x}{4x-1} < \frac{1}{2}$
42.  $5(2x-7) - 3(2x+3) \leq 0$  and  $2x+19 \leq 6x+45$ .

### LONG ANSWER TYPE QUESTIONS (6 MARKS)

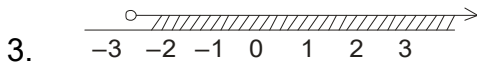
**Solve to the following system of inequalities and represent solution on number line:**

43.  $2x + y \leq 24$ ,  $x + y < 11$ ,  $2x + 5y \leq 40$ ,  $x \geq 0$ ,  $y \geq 0$
44.  $3x + 2y \geq 24$ ,  $3x + y \leq 15$ ,  $x \geq 4$
45.  $x - 2y \leq 3$   
 $3x + 4y > 12$   
 $x \geq 0$ ,  $y \geq 1$

## ANSWERS

1.  $\{1, 2, 3, 4\}$

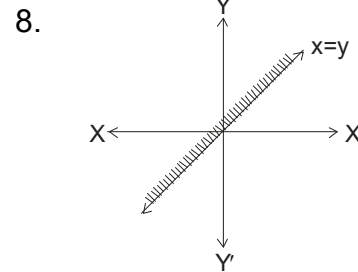
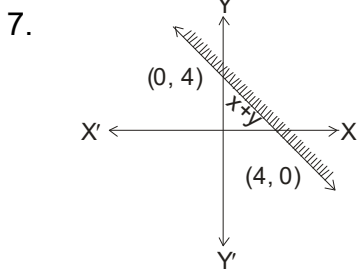
2.  $(-3, \infty)$



4.  $(-\infty, 2)$  or  $x < 2$

5.  $-3 < x < 0$

6.  $\left[\frac{-2}{3}, \frac{5}{3}\right]$



9. (a)  $[-\infty, -3]$

10. (b)

(b)  $[2, \infty]$

11. (a)

(c)  $[-2, 2]$

12. (c)

(d) Slack

13. (d)

(e)  $(-\infty, -5) \cup (5, \infty)$

14. (c)

15. (b)

16. (d)

17. (d)

18. (c)

19. (a) False

(b) True

(c) True

(d) False

20.  $X > 4$
21.  $(-\infty, -3) \cup (2, \infty)$
22.  $\left(-\infty, \frac{63}{10}\right]$
23.  $\left(-\infty, \frac{-13}{2}\right)$
24.  $\left[\frac{-34}{3}, \frac{22}{3}\right]$
25.  $(-\infty, -3) \cup (2, \infty)$
26.  $(-12, -5)$
27.  $(-\infty, 2]$
28.  $(-\infty, -3) \cup (2, \infty)$
29.  $(6, 8)$  and  $(8, 10)$
30. More than 2000 cassettes
31. Between 9.8 m and 13.8 m
32. Between 6.27 and 8.07.
33. Solution set =  $\phi$
34.  $[-4, -1] \cup [5, 7]$
35.  $(4, 9)$
36.  $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$
37.  $(-\infty, -2) \cup (1, \infty)$
38. He must score greater than or equal to 52 and less than 77.
39. Between 100 litre and 150 litre

40.  $(3, \infty)$  

41. Number solution

42.  $[-7, 11]$  