

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

► **Binomial Theorem for Positive Integers :**

- $(x + y)^n = {}^n C_0 y^0 + {}^n C_1 x^{n-1} y^1 + {}^n C_2 x^{n-2} y^2 + \dots$
 $\dots + {}^n C_r x^{n-r} y^r + \dots + {}^n C_n x^0 y^n,$

Where n is any positive integer.

- General Term = $T_{r+1} = {}^n C_r x^{n-r} y^r$, where $0 \leq r \leq n$.
- Total number of terms in expansion $(x + y)^n$ is $n + 1$.

► **Middle Term :**

- If n is even, then there is only one middle term

$$\text{M.T.} = \left(\frac{n+1}{2} \right) \text{th term}$$

- If n is odd, then there are two middle terms

(i) $\text{M.T.} = \left(\frac{n+1}{2} \right) \text{th term}$

(ii) $\text{M.T.} = \left(\frac{n+1}{2} + 1 \right) \text{th term}$

► **Some important observations :**

- In expansion $(x + y)^n$

$$T_{r+1} [(r + 1)^{\text{th}} \text{ term from beginning}] = {}^n C_r x^{n-r} y^r$$

$$T'_{r+1} [(r + 1)^{\text{th}} \text{ term from end}] = {}^n C_{n-r} x^r y^{n-r}$$

- $(x + y)^n = {}^n C_0 x^0 + {}^n C_1 x^1 y^1 + {}^n C_2 x^2 y^2 + \dots + {}^n C_n x^n$.

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write number of terms in the expansion of $\left\{(2x + y^3)^4\right\}^7$.
2. Expand $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$ using binomial theorem.
3. Write value of ${}^{2n-1}C_5 + {}^{2n-1}C_6 + {}^{2n}C_7$ use $[{}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r]$
4. Which term is greater $(1.2)^{4000}$ or 800?
5. Find the coefficient of x^{-17} , in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.
6. Find the sum of the coefficients in $(x + y)^8$
[Hint : Put $x = 1, y = 1$]
7. If ${}^n C_{n-3} = 720$, find n.

11. For all $n \in \mathbb{N}$, $2^{4n} - 15n - 1$ is divisible by -
(a) 125 (b) 225
(c) 450 (d) 625.
12. What is the coefficient of x^n in $(x^2 + 2x)^{n-1}$?
(a) $(n-1)2^{(n-1)}$ (b) $(n-1) \times 2^{(n-1)}$
(c) $(n-1)2^n$ (d) $n \cdot 2^{(n-1)}$.
13. The coefficient of x^{-3} in the expansion of $\left[x - \frac{m}{x}\right]^{11}$ is -
(a) $-924 m^7$ (b) $-792 m^5$
(c) $-792 m^6$ (d) $-330 m^7$.
14. In the expansion of $\left[x^2 - \frac{1}{3x}\right]^9$, the term without x is equal to -
(a) $\frac{28}{81}$ (b) $\frac{-28}{243}$
(c) $\frac{28}{243}$ (d) None of these.
15. If in the expansion of $(1+x)^{20}$, the coefficients of r^{th} and $(r+4)^{\text{th}}$ term are equal, then x is equal to -
(a) 7 (b) 8
(c) 9 (d) 10.
16. If in the expansion of $(1+x)^5$, the coefficients of $(r-1)^{\text{th}}$ and $(2r+3)^{\text{th}}$ terms are equal, then the value of x -
(a) 5 (b) 6
(c) 4 (d) 3.
-

17. The total number of terms in expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is -
- (a) 202 (b) 51
(c) 50 (d) None of these.
18. The middle term in the expansion of $\left[\frac{2x}{3} - \frac{3}{2x^2}\right]^{2n}$ is -
- (a) ${}^{2n}C_n$ (b) $(-1)^n {}^{2n}C_n x^{-n}$
(c) ${}^{2n}C_n x^{-n}$ (d) None of these.
19. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the same, then the value of a is -
- (a) $\frac{-7}{9}$ (b) $\frac{-9}{7}$
(c) $\frac{7}{9}$ (d) $\frac{9}{7}$.

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

20. How many term are free from radical signs in the expansion of $\left(x^{\frac{1}{5}} + y^{\frac{1}{10}}\right)^{55}$.
21. Find the constant term in expansion of $\left(x - \frac{1}{x}\right)^{10}$.
22. Find the value of
- $$\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \times 6 - {}^8C_3 \times 6^2 + \dots + {}^8C_8 6^7$$

23. Find 4th term from end in the expansion of find the value of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.
24. Find middle term in the expansion of $(x - 2y)^8$.
25. Which term is independent of x in the expansion of $\left(3x^3 - \frac{1}{2x^3}\right)^{10}$.
26. Find the 11th term from end in expansion of $\left(2x - \frac{1}{x^2}\right)^{25}$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Section - C

27. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.
28. In $\left(3x^2 - \frac{1}{x}\right)^{18}$ which term contains x^{12} .
29. In $\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2x^2}}\right)^{10}$ find the term independent of x.
30. Evaluate $(\sqrt{2} + 1)^5 - (\sqrt{2} - 1)^5$ using binomial theorem.
31. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4 .
32. Find the coefficients of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

33. Show that $3^{2n+2} - 8n - 9$ is divisible by 8.
34. If the term free from x in the expansion of $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405. Find the value of k .
35. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1024}$.
36. If for positive integers $r > 1$, $n > 2$ the coefficients of the $(3r)^{\text{th}}$ term and $(r + 2)^{\text{th}}$ powers of x in the expansion of $(1 + x)^{2n}$ are equal, then prove that $n = 2r + 1$.
37. If a , b , c and d in any binomial expansion be the 6^{th} , 7^{th} , 8^{th} and 9^{th} terms respectively, then prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$.
38. If in the expansion of $(1 + x)^n$, the coefficients of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.
39. Show that $2^{4n+4} - 15n - 16$ where $n \in \mathbb{N}$ is divisible by 225.
40. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio $1 : 3 : 5$, then show that $n = 7$.
41. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$.
42. Find the value of r , if the coefficient of $(2r + 4)^{\text{th}}$ term and $(r - 2)^{\text{th}}$ term in the expansion of $(1 + x)^{18}$ are equal.

43. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.
44. The coefficient of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 6 : 30. Find n.

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

45. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6(1 - x)^7$ is 171.
46. If the 3rd, 4th and 5th terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n.
47. If the coefficients of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ are equal, then show that $ab = 1$.
48. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7th term from the beginning to the 7th term from the end is 1:6, find n.
49. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms in the expansion of $(1 + x)^n$
- Prove that $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$.
50. Using binomial theorem, find the remainder when 5^{103} is divided by 13.
51. Find the remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9.

52. Find the coefficient of x^n in expansions of $(1 + x)(1 - x)^n$.
53. Find the value of $(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6$ and show that $(\sqrt{2} + 1)^6$ lies between 197 and 198.
54. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3}x\right)^9$.
55. If the coefficients of r^{th} , $(r + 1)^{\text{th}}$ and $(r + 2)^{\text{th}}$ terms in the expansion of $(1 + x)^4$ are in A.P find the value of r .
56. If the expansion of $(1 - x)^{2n-1}$, the coefficients of x^r is denoted by a_r , then prove $a_{(r-1)} + a_{(2n-r)} = 0$.
57. If the coefficient of 5^{th} , 6^{th} and 7^{th} terms in the expansion of $(1 + x)^n$ are in A.P., then find the value of n .
58. Find the coefficient of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ and find the relation between a and b so that these coefficients are equal.
59. The coefficients of 2^{nd} , 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in A.P. Prove that $2n^2 - 9n + 7 = 0$.
60. Show that the middle term in the expansion of $\left[x - \frac{1}{x}\right]^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} (-2n)^n$.

ANSWERS

1. 29

2. $\frac{x^3}{a^3} - \frac{6x^2}{a^2} + 15\frac{x}{a} - 20 + 15\frac{a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. -1365

6. 256

7. $n = 10$

8. (a) 18

9. (a) False

(b) ${}^9C_3 \times \left(\frac{-1}{3}\right)^3$

(b) True

(c) ${}^{10}C_5$

(c) True

(d) 3

(d) False

(e) 152

(e) False

(f) $\frac{672}{x^3}$

(f) True

10. (b)

11. (b)

12. (a)

13. (d)

14. (c)

15. (c)

16. (a)

17. (b)

18. (b)

19. (d)

20. 6 terms (0, 10, 20, 30, 40, 40, 50)

21. $-252 = -{}^{10}C_5$

22. ${}^{31}C_6 - {}^{21}C_6 / \frac{5^8}{6}$

23. $\frac{672}{x^3}$

24. $1120 x^4 y^4$

25. $\frac{-15309}{8}$

26. ${}^{25}C_{15} \times \frac{2^{10}}{x^{20}}$

27. $a = 3, b = 2, n = 3$

28. 9th term

29. $T_3 = \frac{5}{6}$

30. 82

31. 28

32. 10

34. $k = \pm 3$

35. 129 integral terms

36. $x = \frac{1}{\sqrt{10}}$ or 100

38. $n = 8, 4^{\text{th}}, 5^{\text{th}}$ and 6^{th}

42. $r = 6$

43. $\left(2x^2 - \frac{3}{x}\right)^{11}$

44. $n = 41$

46. $a = 2, x = 1, n = 7$

48. 9

50. 8

51. 2

52. $(-1)^n [1 - n]$

53. Zero

54. $\frac{17}{54}$

55. 5

57. $n = 7$ or 14

58. $ab = 1$