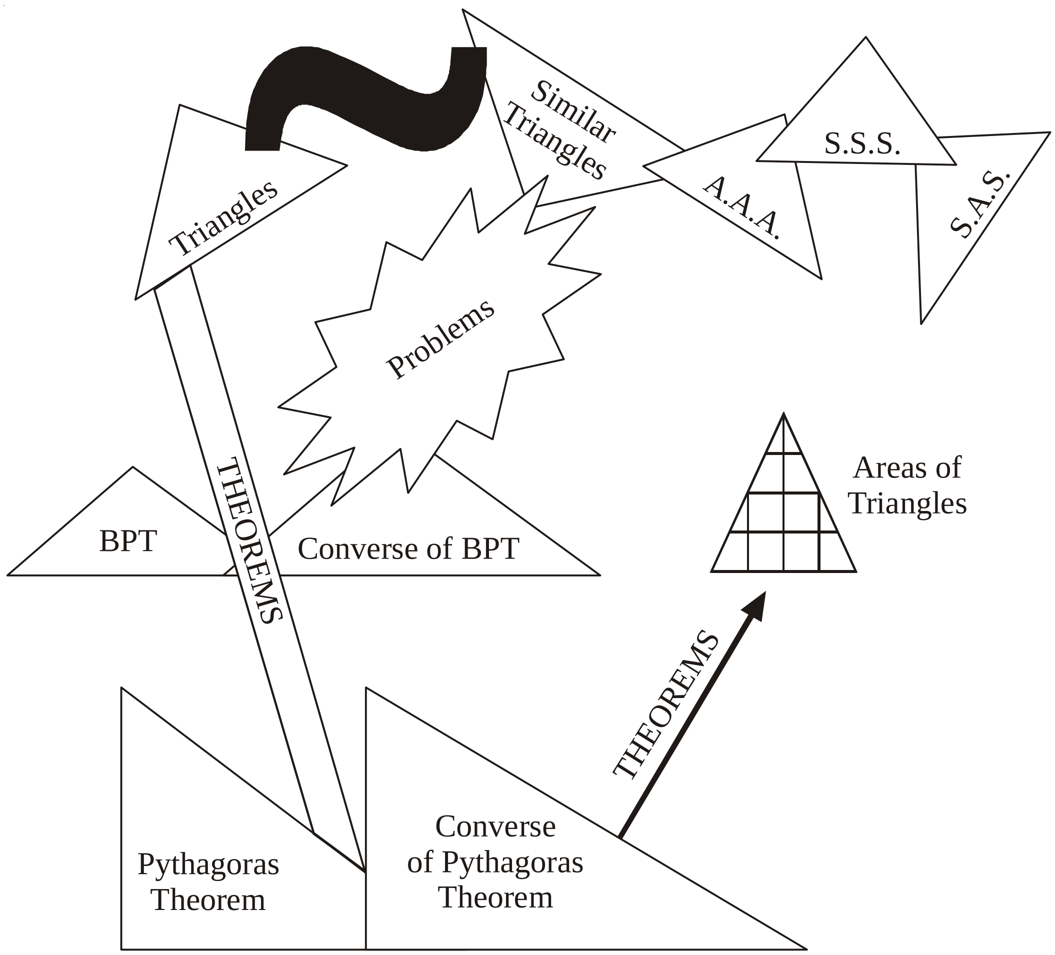
CHAPTER

6 Triangles



Key Points:

1. **Similar Triangles:** Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides ar proportional.
2. **Criteria for Similarity:**

   in ΔABC and ΔDEF

   (*i*) **AAA Similarity :** $\Delta ABC \sim \Delta DEF$ when $\angle A, \angle D, \angle B = \angle E$ and $\angle C = \angle F$

   (*ii*) SAS Similarity :

   $$\Delta ABC \sim \Delta DEF \text{ when } \frac{AB}{DE} = \frac{BC}{EF} \text{ and } \angle B = \angle E$$

   (*iii*) SSS Similarity : $\Delta ABC \sim \Delta DEF, \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

3. **The proof of the following theorems can be asked in the examination :**

   (*i*) Basic Proportionality Theorem : If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

   (*ii*) The rato of aras of two similar triangles is equal to the square of the ratio of their corresponding sides.

   (*iii*) Pythagoras Theorem: In a right triangles the square of the hypotenuse is equal to the sum of the squares of the other two sides.

   (*iv*) Converse of pythagoras theorem : In a triangle, if the square of one side is equal to the sum of squares of other sides then the angle opposite to the first side is a right angle.

VERY SHORT ANSWER TYPE QUESTIONS

1. **Fill in the blanks :**

   (*i*) All equilateral triangles are ___________ .

   (*ii*) If $\Delta ABC \sim \Delta FED$, then $\frac{AB}{\_\_\_} = \frac{\_\_\_}{ED}$.

   (*iii*) Circles with equal radii are __________ .

   (*iv*) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the __________ ratio.

   (*v*) In ___________ triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

2. **State True or False :**

(*i*) All the similar figures are always congruent.

(*ii*) The Basic Proportionality Theorem was given by Pythagoras.

(*iii*) The mid-point theorem can be proved by Basic Proprotionality Theorem.

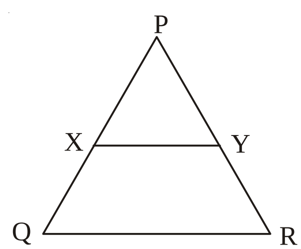
(*iv*) Pythagoras Theorem is valid for right angled triangle.

(*v*) If the sides of two similar triangles are in the ratio 4 : 9, then the areas of these triangles are in the ratio 16 : 81.

3. **Match the following :**

Column I	Column II
(*a*) If corresponding angles are equal in two triangles, then the two triangles are similar.	(*i*) SAS similarity criterion
(*b*) If sides of one triangle are proportional to the sides of the other triangle, then the two triangles are similar.	(*ii*) ASA similarity criterion
(*c*) If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.	(*iii*) AAA similarity criterion
	(*iv*) SSS similarity criterion

4. In the following figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



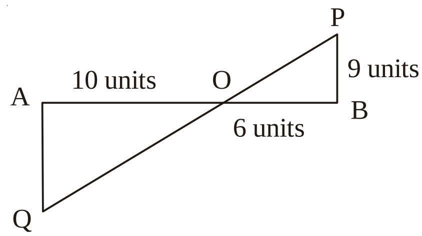
(*a*) XY = QR

(*b*) $XY = \frac{1}{3}QR$

(*c*) $XY^2 = QR^2$

(*d*) $XY = \frac{1}{2}QR$

5. In the following figure, QA ⊥ AB and PB ⊥ AB, then AQ is



(*a*) 15 units (*b*) 8 units

(*c*) 5 units (*d*) 9 units

6. The ratio of areas of two similar triangles is equal to the

(*a*) ratio of their corresponding sides.

(*b*) ratio of their corresponding altitudes.

(*c*) ratio of the square of their corresponding sides.

(*d*) ratio of their perimeter.

7. The areas of two similar triangles are 144 cm^2 and 81 cm^2. If one median of the first triangleis 16 cm, length of corresponding median of the second triangle is

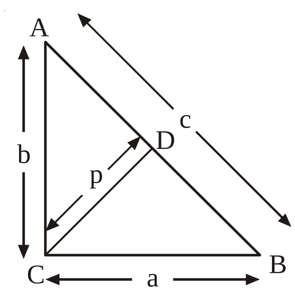
(*a*) 9 cm (*b*) 27 cm

(*c*) 12 cm (*d*) 16 cm

8. In a right triangle ABC, in which $\angle C = 90°$ amd $CD \perp AB$. If $BC = a$, $CA = b$, $AB = c$ and $CD = p$, then

(*a*) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (*b*) $\frac{1}{p^2} \neq \frac{1}{a^2} + \frac{1}{b^2}$

(*c*) $\frac{1}{p^2} < \frac{1}{a^2} + \frac{1}{b^2}$ (*d*) $\frac{1}{p^2} > \frac{1}{a^2} + \frac{1}{b^2}$



9. If $\Delta ABC \sim \Delta DEF$, ar($\Delta$DEF) = 100 cm^2 and $\frac{AB}{DE} = \frac{1}{2}$, then ar(DABC) is

(*a*) 50 cm^2 (*b*) 25 cm^2

(*c*) 4 cm^2 (*d*) 200 cm^2

10. If the three sides of a triangle are a, $\sqrt{3}a$ and $\sqrt{2}a$, then the measure of hte angle opposite to longest side is

(*a*) 45° (*b*) 30°

(*c*) 60° (*d*) 90°

11. A vertical pole of length 3 m casts a shadow of 7 m and a tower casts a shadow of 28 m at a time. The height of tower is (*a*) 10 m (*b*) 12 m

(*c*) 14 m (*d*) 16 m

12. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is **(NCERT Exempler)**

(*a*) 9 cm (*b*) 10 cm

(*c*) 8 cm (*d*) 20 cm

13. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true? **(NCERT Exempler)**

(*a*) BC.EF = AC.FD (*b*) AB.EF = AC.DE

(*c*) BC.DE = AB.EF (*d*) BC.DE = AB.FD

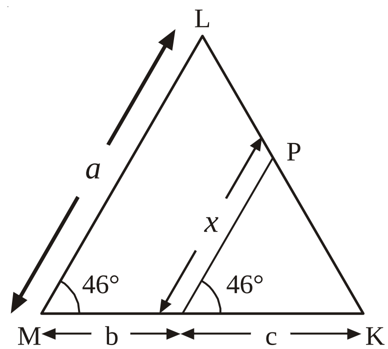
14. Write the statement of pythagoras theorem.

15. Write the statement of Basic Proportionality Theorem.

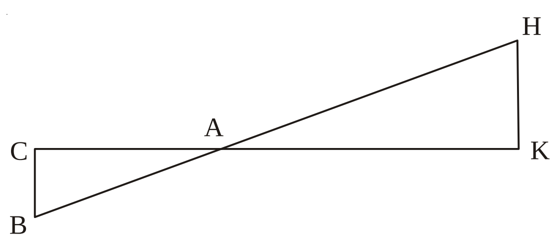
16. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle?

17. If ΔABC ~ ΔQRP, $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{9}{4}$, AB = 18 cm, BC = 15 cm, then find the length of PR. **(CBSE 2018)**

18. In the given Fig., ∠M = ∠N = 46°, Express *x* in terms of *a*, *b* and *c*.



19. In the given Fig. ΔAHK ~ ΔABC. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. **(CBSE 2010)**



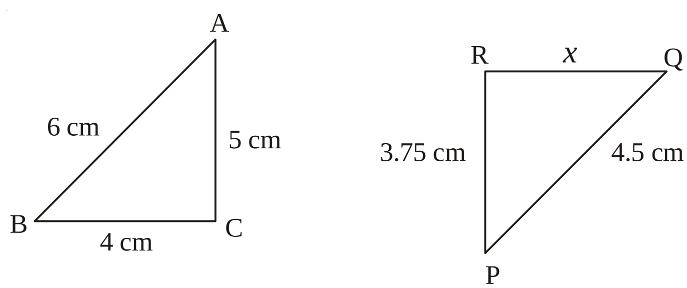
20. It is given that ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠F = ∠P?

21. If the corresponding Medians of two similar triangles are in the ratio 5 : 7. Then find the ratio of their sides.

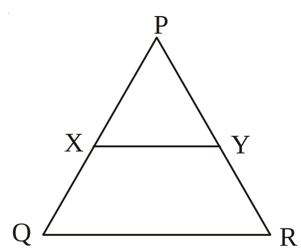
22. An aeroplane leaves an airport and flies due west at a speed of 2100 km/hr. At the same time, another aeroplane leaves the same place at airport and flies due south at a speed of 2000 km/hr. How far apart will be the two planes after 1 hour?

23. The areas of two similar ΔABC and ΔDEF are 225 cm^2 and 81 cm^2 respectively. If the longest side of the larger triangle ΔABC be 30 cm, find the longest side of the smaller triangle DEF.

24. In the given figure, if $\Delta ABC \sim \Delta PQR$, find the value of x?

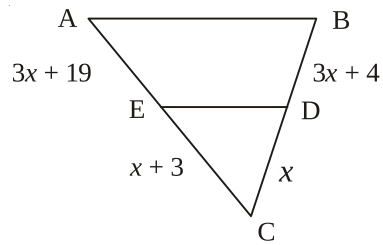


25. In the given figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, find XY : QR.



26. In the given figure, find the value of x which will make DE || AB ?

(NCERT Exempler)



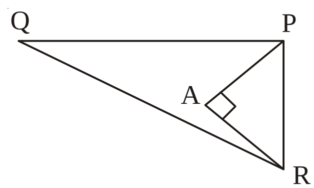
27. If ΔABC ~ ΔDEF, BC = 3EF and ar (DABC) = 117cm^2 find area (ΔDEF).

28. If ΔABC and ΔDEF are similar triangles such that ∠A = 45° and ∠F = 56°, then find the ratio of their corresponding attitudes.

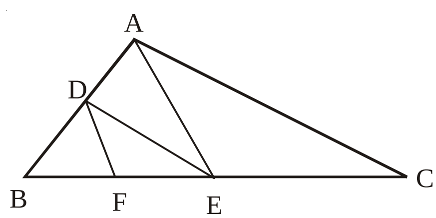
29. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then find the ratio of their corresponding attitudes.

SHORT ANSWER TYPE QUESTIONS-I

30. In the given Fig. PQ = 24 cm, QR = 26 cm, ∠PAR = 90°, PA = 6 cm and AR = 8 cm, find ∠QPR.

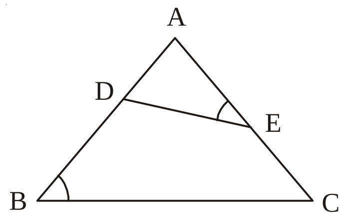


31. In the given Fig., DE || AC and DF || AE. Prove that $\frac{FE}{BF} = \frac{EC}{BE}$

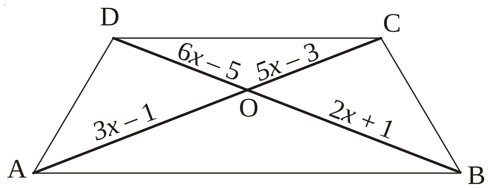


32. In ΔABC, AD ⊥ BC. Such that AD^2 = BD × CD. Prove that ΔABC is right angled triangle.

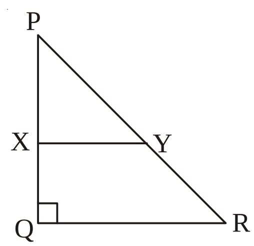
33. In the given Fig., D and E are points on sides AB and CA of ΔABC such that ΔB = ∠AED. Show that ΔABC ~ ΔAED.



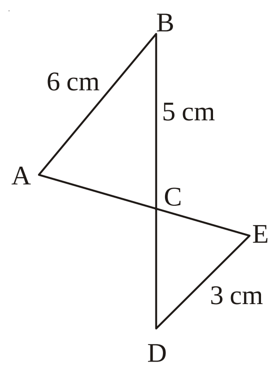
34. In the given fig., AB || DC and diagonals AC and BD intersects at O. If OA = $3x - 1$ and OB = $2x + 1$, OC = $5x - 3$ and OD = $6x - 5$, find the value of x.



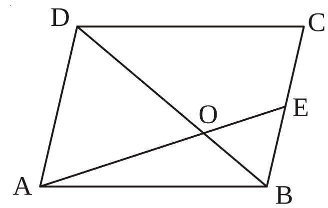
35. In the given Fig. PQR is a triangle, right angled at Q. If XY || QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.



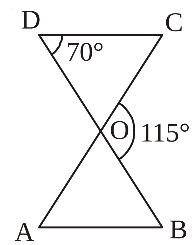
36. In the given figure, AB || DE. Find the length of CD.



37. In the given figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm find BC.



38. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 70$. Find, (*i*) $\angle DOC$, (*ii*) $\angle DCO$, (*iii*) $\angle OAB$, (*iv*) $\angle OBA$.

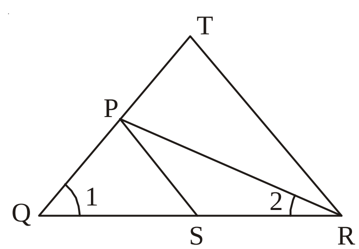


39. Perimeter of two equilateral triangles ABC and PQR are 144 m and 96 m, Find ar (ΔABC) : ar (ΔPQR).

SHORT ANSWER TYPE QUESTIONS-II

40. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ then prove that $\Delta PQS \sim \Delta TQR$.

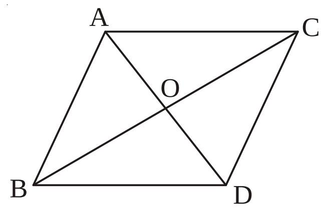
(NCERT)



41. In equilateral ΔABC, $AD \perp BC$. Prove that $3BC^2 = 4AD^2$.

42. In ΔABC, $\angle ACB = 90°$ and $CD \perp AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$. (HOTS)

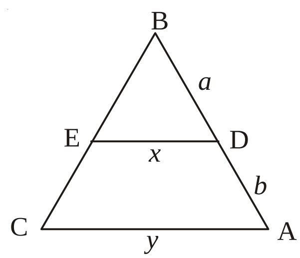
43. In the adjoining figure ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DBC)} = \frac{AO}{DO}$.



44. If AD and PS are medians of ΔABC and ΔPQR respectively where $\Delta ABC \sim \Delta PQR$, Prove that $\frac{AB}{PQ} = \frac{AD}{PS}$.

45. In the given figure, DE || AC. Which of the following is correct?

$x = \frac{a+b}{ay}$ or $x = \frac{ay}{a+b}$

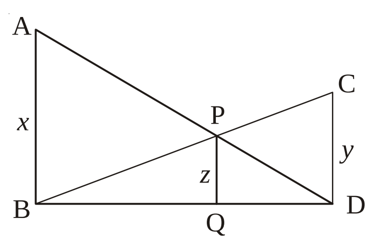


46. Prove that the sum of the square of the sides of a rhombus is equal to the sum of the squares of its diagonals. **(NCERT, CBSE 2019)**

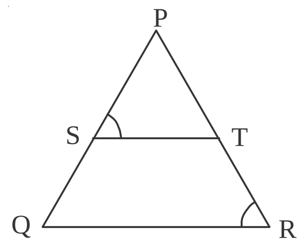
47. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how for she is away from the base of the pole. **(NCERT Exempler)**

48. Two poles of height *a* metrs and *b* metres are *p* metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is gives by $\frac{ab}{a+b}$ metres.

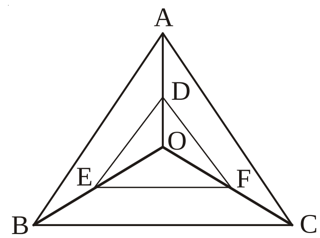
49. In the given figure AB || PQ || CD, AB = x, CD = y and PQ = z. Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



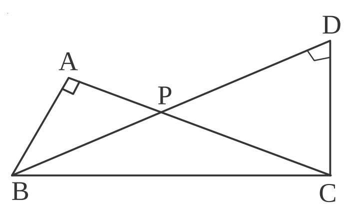
50. In the given figure $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isoscles triangle. **(NCERT)**



51. In the figure, a point O inside ΔABC is joined to its vertices. From a point D on AO, DE is drawn parallel to AB and from a point E on BO, EF is drawn parallel to BC. Prove that DF || AC.



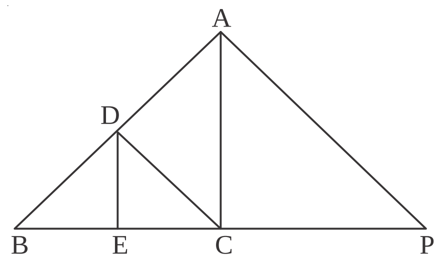
52. Two triangles BAC and BDC, right angled at A and D respectively are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P. PRove that AP × PC = DP × PB. **(CBSE 2019)**



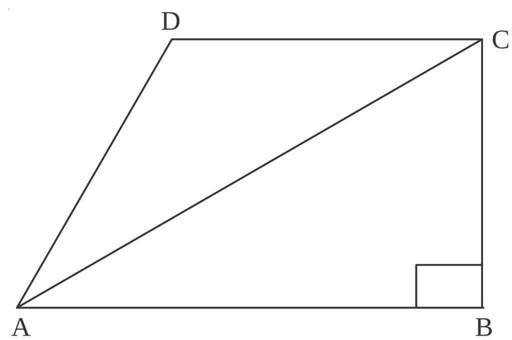
53. Hypotenuse of a right triangle is 25 cm and out of the remaining two sides, one is larger than the other by 5 cm, find the lenghts of the other two sides.

(NCERT Exempler)

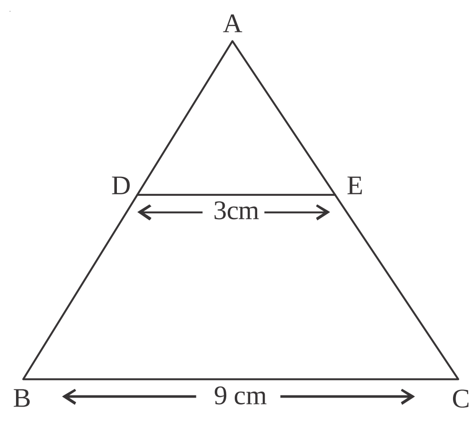
54. In the given figure DE || AC and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that DC || AP.



55. In a quadrilateral ABCD, $\angle B = 90°$, $AD^2 = AB^2 + BC^2 + CD^2$. Prove that $\angle ACD = 90°$.



56. In the given figure, DE || BC, DE = 3 cm, BC = 9 cm and ar (DADE) = 30 cm^2. Find ar (BCED).



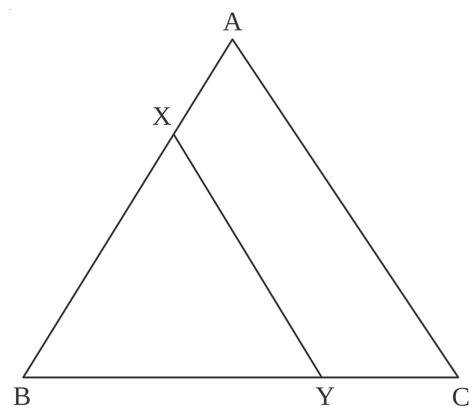
57. In an equilateral ΔABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. **(NCERT, CBSE 2018)**

58. In ΔPQR, $PD \perp QR$ such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$ and a, b, c, d are positive units. Prove that $(a + b)(a - b) = (c + d)(c - d)$.

(NCERT Exempler)

59. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. **(CBSE 2010, 2018, 2019)**

60. In the given figure, the line segment XY is Parallel to AC of ΔABC and it divides the triangle into two parts of equal areas. Prove that $\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$.



61. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that $\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$.

62. Prove that if in a triangle, the square on one side is equal to the sum of the squares on the other two sides, then the angle opposite to the first side is a right angle.

(CBSE 2019)

63. Prove that is a right angle triangle, the square of the hypotenuse is equal the sum of the squares of other two sides. **(CBSE 2018, 2019)**

64. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

(CBSE 2019)

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. (*i*) Similar (*ii*) $\frac{AB}{FE} = \frac{BC}{ED}$ (*iii*) Congruent

(*iv*) Same (*v*) Right

2. (*i*) False (*ii*) False (*iii*) True

(*iv*) True (*v*) True

3. (*a*) (*iii*) AAA similarity criterion.

(*b*) (*iv*) SSS similarity criterion.

(*c*) (*i*) SAS similarity criterion.

4. (B) $XY = \frac{1}{3}QR$

5. (A) 15 units

6. (C) Ratio of the square of their corresponding sides.

7. (C) 12 cm

8. (A) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. (B) 25 cm^2

10. (D) 90°

11. (B) 12 m

12. (B) 10 cm

13. (C) BC.DE = AB.EF

16. No, because $(12)^2 + (16)^2 \neq (18)^2$

17. 10 cm

18. $\Delta KPN \sim \Delta KLM$

$$\frac{x}{a} = \frac{c}{b+c}$$

$$x = \frac{ac}{b+c}$$

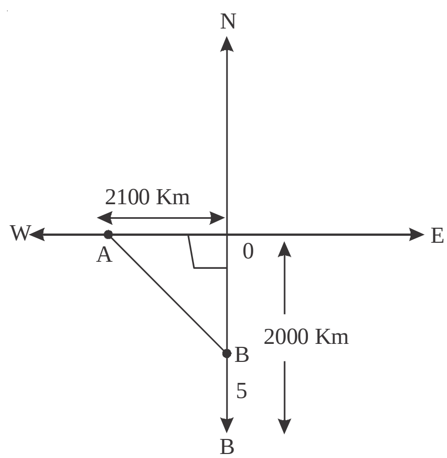
19. $\frac{AK}{AC} = \frac{HK}{BC} \Rightarrow \frac{10}{AC} = \frac{7}{3.5} \Rightarrow AC = 5$ cm

20. $\angle D = \angle R$ (True)

$\angle F = \angle P$ (False)

21. 5 : 7

22.



$AB = \sqrt{(2100)^2 + (2000)^2} = 2900$ km

23. Let longest side of the ΔDEF be x cm.

$$\frac{225}{81} = \left(\frac{30}{x}\right)^2$$

$x = 18$ cm

24. $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{6}{4.5} = \frac{4}{x} \Rightarrow x = 3$cm

25. $\Delta PXY \sim \Delta PQR$

$$\frac{PX}{PQ} = \frac{XY}{QR} = \frac{1}{3}$$

$\therefore XY : QR = 1 : 3$

26. $\frac{x+3}{3x+19} = \frac{x}{3x+4}$ (By B.P.T.)

$x = 2$

27. $\frac{ar\,(ABC)}{ar\,(DEF)} = \left(\frac{BC}{EF}\right)^2 = \left(\frac{3EF}{EF}\right)^2 = \left(\frac{3}{1}\right)^2$

$$\frac{117}{ar(DEF)} = 9 \quad \Rightarrow \quad ar(DEF) = 13 \text{ cm}^2$$

28. $\angle F = \angle C = 56°$

29. 2 : 3

30. $PR = \sqrt{(6)^2 + (8)^2} = 10$ cm.

As $QR^2 = PQ^2 + PR^2$, therefore $\angle QPR = 90°$.

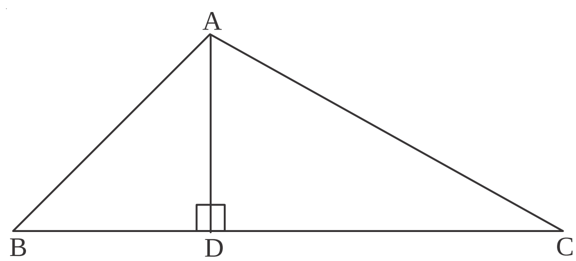
31. DE || AC, $\frac{AD}{DB} = \frac{EC}{BE}$ ...(1) [∵ BPT]

DF || AE, $\frac{AD}{DB} = \frac{FE}{BF}$ ...(2) [∵ BPT]

From (1) and (2), we get

$$\frac{FE}{BF} = \frac{EC}{BE}$$

32. In ΔADC, $AD^2 = AC^2 - DC^2$ ...(1)



In ΔADB, $AD^2 = AB^2 - BD^2$ ...(2)

Adding (1) and (2), we have

$2AD^2 = AC^2 + AB^2 - BD^2 - DC^2$

$2AD^2 + BD^2 + DC^2 = AC^2 + AB^2$

$2BD \times CD + BD^2 + DC^2 = AC^2 + AB^2$

$(BD \times DC)^2 = AC^2 + AB^2$

$BC^2 = AC^2 + AB^2$

By converse of Pythagoras Theorem, ΔABC is a right angled triangle.

33. $\angle B = \angle AED$ (Given)

$\angle A = \angle A$ (Common)

$\therefore\ \Delta ABC \sim \Delta AED$ [AA similarity criterion]

34. $\frac{3x-1}{5x-3} = \frac{2x+1}{6x-5} \Rightarrow x = \frac{1}{2}$ or 2

But $x = \frac{1}{2}$ is neglected due $(5x-3)$ get negative value.

So, $x = 2$ is the required value.

35. $\frac{PX}{XQ} = \frac{PY}{YR} \Rightarrow \frac{1}{2} = \frac{4}{YR} \Rightarrow YR = 8$ cm

$\therefore$ PR = 8 + 4 = 12 cm

QR = $\sqrt{(12)^2-(6)^2} = 6\sqrt{3}$ cm

36. $\Delta ABC \sim \Delta EDC$ (AA Similarity criterion)

$\frac{6}{3} = \frac{5}{CD}$

CD = 2.5 cm

37. $\Delta BOE \sim \Delta DOA$ (AA Similarity criterion)

$\frac{BO}{DO} = \frac{BE}{DA}$

$\frac{1}{2} = \frac{1.5}{DA}$

DA = 3 cm

BC = DA = 3 cm (Opposite sides of a parallelogram)

38. (*i*) 65°

(*ii*) 45°

(*iii*) 45°

(*iv*) 70°

39. $\frac{ar\,(\Delta\,ABC)}{ar\,(\Delta\,PQR)} = \left(\frac{144}{96}\right)^2 = \frac{9}{4}$

$\therefore$ ar (ΔABC) : ar (ΔPQR) = 9 : 4

40. In ΔPQR, $\angle 1 = \angle 2$

PR = PQ [Opposite sides of equal angles]

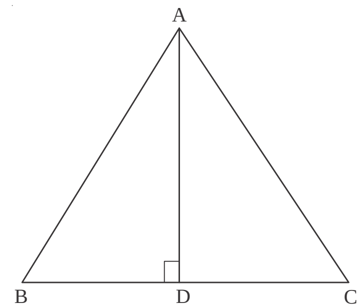
$\therefore \frac{QR}{QS} = \frac{QT}{PQ}$ and $\angle 1 = \angle 1$ (Common)

$\therefore \Delta PQS \sim \Delta TQR$ (SAS Similarity criterion)

41. $\Delta ADB \cong \Delta ADC$

BD = DC

$\therefore BD = \frac{1}{2} BC$ ...(1)



In right angled ΔADB,

$AB^2 = AD^2 + BD^2$

$BC^2 = AD^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ AB = BC = CA and from (1)]

$3BC^2 = 4AD^2$

42. $\Delta ABC \sim \Delta CBD$

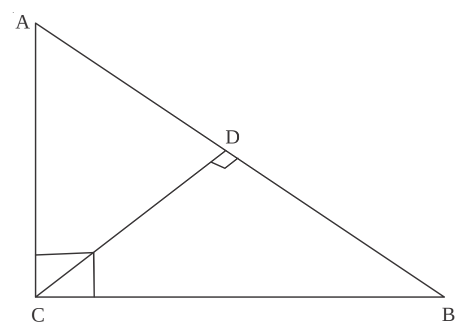
$\therefore BC^2 = AB.BD$ ...(1)

$\Delta ABC \sim \Delta ACD$

$\therefore AC^2 = AB.AD$ ...(2)

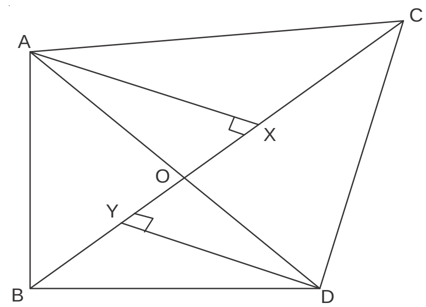
Divide (1) by (2), we get

$\frac{BC^2}{AC^2} = \frac{BD}{AD}$



43. Draw $AX \perp BC$ and $DY \perp BC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AX}{\frac{1}{2} \times BC \times DY} = \frac{AX}{DY} \quad ...(1)$$



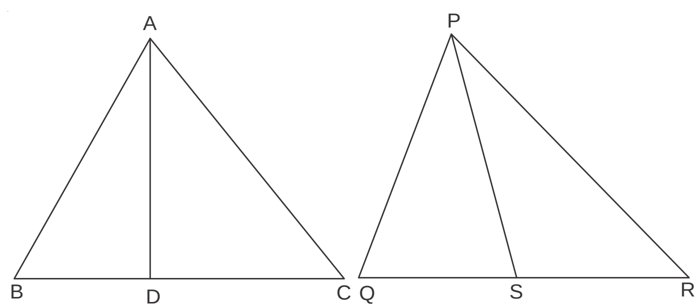
$\Delta AXO \sim \Delta DYO$ [AA similarity criterion]

$\frac{AX}{DY} = \frac{AO}{DO}$ ...(2) (C.P.S.T.)

From (1) and (2), we get

$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

44.



As $\Delta ABC \sim \Delta PQR$, Hence $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QS}$

In ΔABD and ΔPQS

$\frac{AB}{PQ} = \frac{BD}{QS}$ and $\angle B = \angle Q$.

$\therefore$ $\Delta ABD \sim \Delta PQS$ (SAS Similarity criterion).

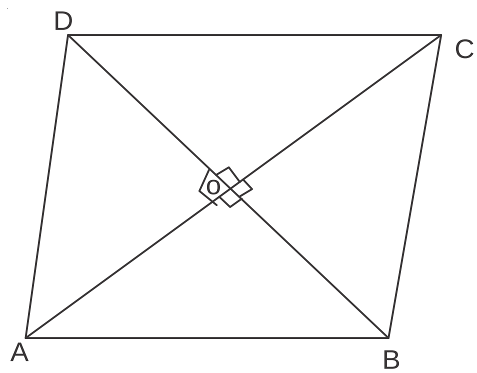
Hence, $\frac{AB}{PQ} = \frac{AD}{PS}$ (C.P.S.T.)

45. $\Delta BED \sim \Delta BCA$

$$\frac{x}{y} = \frac{a}{a+b}$$

$$\Rightarrow x = \frac{ay}{a+b}$$

46.



In right angled ΔAOB, $AB^2 = OA^2 + OB^2$ ...(1)

In right angled ΔBOC, $BC^2 = OB^2 + OC^2$ ...(2)

In right angled ΔCOD, $CD^2 = OC^2 + OD^2$ ...(3)

In right angled ΔDOA, $DA^2 = OD^2 + OA^2$ ...(4)

Adding (1), (2), (3) and (4), we get

$$AB^2 + BC^2 + CD^2 + DA^2 = 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$$

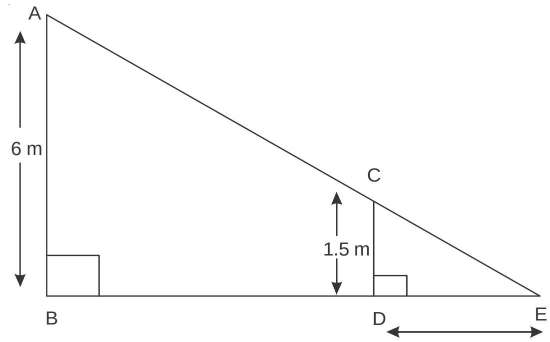
$$= 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2 + 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2$$

[$\because$ Diagonals of rhombus $\perp$ bisect each other]

$$= AC^2 + BD^2$$

47. $\Delta ABE \sim \Delta CDE$

$$\frac{AB}{CD} = \frac{BE}{DE}$$



$$\frac{6}{1.5} = \frac{3 + BD}{3}$$

$BD = 9m$

48. **To prove :** $EF = \dfrac{ab}{a+b}$

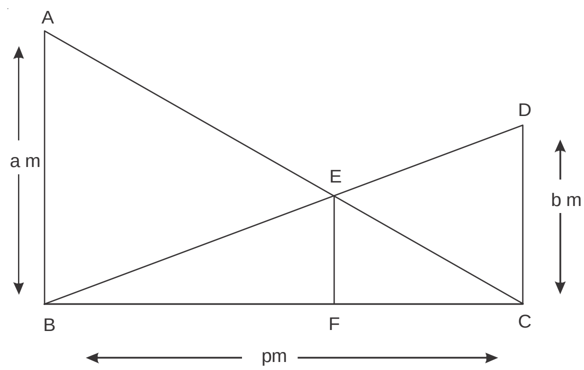
Proof : AB || EF || DC

$\Delta EFC \sim \Delta ABC$

$$\frac{EF}{AB} = \frac{FC}{BC} \quad ...(1)$$

$\Delta BFE \sim \Delta BCD$

$$\frac{EF}{CD} = \frac{BF}{BC} \quad ...(2)$$



Adding (1) and (2), we get

$$\frac{EF}{AB} + \frac{EF}{CD} = \frac{FC + BF}{BC}$$

$$EF\left[\frac{1}{AB} + \frac{1}{CD}\right] = \frac{BC}{BC}$$

$$EF\left[\frac{1}{a} + \frac{1}{b}\right] = 1$$

$$EF = \frac{ab}{a+b}$$

49. Same as Q. 48.

50. $\dfrac{PS}{SQ} = \dfrac{PT}{TR}$

By converse of BPT, ST || QR

$\therefore \quad \angle PQR = \angle PST$ (A.I.A)

But $\angle PST = \angle PRQ$

So, $\angle PQR = \angle PRQ$

$\therefore \quad PQ = PR$

So, ΔPQR is an isosceles triangle.

51. In ΔOAB, $\frac{OD}{DA} = \frac{OE}{EB}$ .... (1) ($\because$ BPT)

In ΔOBC, $\frac{OE}{EB} = \frac{OF}{FC}$ .... (2) ($\because$ BPT)

From (1) and (2), we get

$$\frac{OD}{DA} = \frac{OF}{FC}$$

By converse of BPT, DF || AC.

52. ΔAPB ~ ΔDPC (AA Similarity criterion)

$\frac{AP}{DP} = \frac{PB}{PC}$ ($\because$ C.P.S.T.)

AP.PC = DP.PB

53. Let sides of right angled triangle other than hypotenuse be x cm and $(x + 5)$ cm.

By Pythagoras theorem,

$(x)^2 + (x + 5)^2 = (25)^2$

$x = 15$ or -20

But side is always positive, So, $x = 15$.

$\therefore$ Length of two sides is 15 cm and 20 cm.

54. Same as Q.31.

55. In right angled ΔABC, $AC^2 = AB^2 + BC^2$ ...(1)

Given, $AD^2 = (AB^2 + BC^2) + CD^2$

$\Rightarrow \quad AD^2 = AC^2 + CD^2$ [From (1)]

By converse of Pythagoras theorem, $\angle ACD = 90°$.

56. ΔADE ~ ΔABC

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{30}{ar(\Delta ABC)} = \left(\frac{3}{9}\right)^2$$

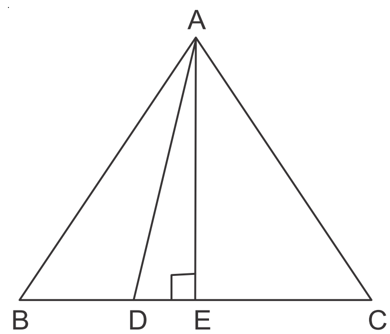
$\therefore$ ar(ΔABC) = 270 cm^2

ar (BCFD) = ar (ΔABC) – ar (ΔADE)

= 270 – 30 = 240 cm^2

57. Draw AE $\perp$ BC

$\Delta ABE \cong \Delta ACE$



$\therefore$ BE = CE $\Rightarrow$ BE = $\frac{1}{2}$ BC

In right angled ΔAED, $AE^2 = AD^2 - DE^2$ ...(1)

In right angled ΔAEB, $AE^2 = AB^2 - BE^2$ ...(2)

From (1) and (2), we have

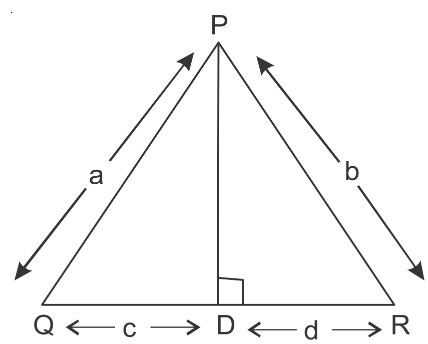
$AD^2 - DE^2 = AB^2 - BE^2$

$$AD^2 - (BE - BD)^2 = BC^2 - \left(\frac{1}{2}BC\right)^2$$

$$AD^2 - \left[\frac{1}{2}BC - \frac{1}{3}BC\right]^2 = BC^2 - \frac{BC^2}{4}$$

$9AD^2 = 7AB^2$

58. In right angled ΔPDQ,



$PD^2 = a^2 - c^2$ ...(1)

In right angled ΔPDR

$PD^2 = b^2 - d^2$ ...(2)

From (1) and (2), we have

$a^2 - c^2 = b^2 - d^2$

$a^2 - b^2 = c^2 - d^2$

$(a - b)(a + b) = (c + d)(c - d)$

59. Theorem 6.6 of NCERT.

60. Given, ar ΔBXY = ar AXYC

ar (Δ ABC) = ar Δ BXY + ar AXYC

= 2 ar ΔBXY

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta BXY)} = \frac{2}{1}$$

$\Delta ABC \sim \Delta XBY$

$$\left(\frac{AB}{XB}\right)^2 = \frac{ar\,(\Delta ABC)}{ar\,(\Delta BXY)}$$

$$\frac{AB}{XB} = \sqrt{2}$$

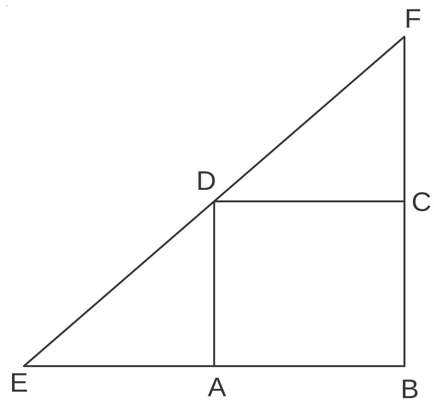
$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

61. $\Delta EAD \sim \Delta EBF$



$$\frac{EA}{EB} = \frac{AD}{BF}$$

$$\Rightarrow \frac{BF}{BE} = \frac{AD}{AE} = \frac{BF - AD}{BE - AE} = \frac{BF - BC}{BA} = \frac{CF}{DC}$$

62. Theorem 6.9 of NCERT.

63. Theorem 6.8 of NCERT.

64. Theorem 6.9 of NCERT.

PRACTICE-TEST

Triangles

Time : 1 Hrs. ***M.M. : 20***

SECTION - A

1. If sides of two similar triangles are in the ratio of 8:10, then areas of these triangles are in the ratio ___________ . **1**

2. If in two triangles ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$, then **1**

(A) ΔPQR ~ ΔCAB (B) ΔPQR ~ ΔABC

(C) ΔCBA ~ ΔPQR (D) ΔBCA ~ ΔPQR

3. ΔABC is an isosceles right triangle, right angled at C, then AB^2 = ...

(A) AC^2 (B) $2\ AC^2$

(C) $4\ AC^2$ (D) $3\ AC^2$ **1**

4. A line DE is drawn parallel to base BC of ΔABC, meeting AB in D and AC at E.

If $\frac{AB}{BD} = 4$ and CE = 2 cm, find the length of AE.

SECTION B

5. The length of the diagonal of a rhombus field are 32 m and 24 m. Find the length of the side of the field. **2**

6. A man goes 24 m towards West and then 10 m towards North. How far is he from the starting point? **2**

7. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. **2**

SECTION C

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersect CD at F. Show that ΔABE ~ ΔDCB. **3**

9. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitude. **3**

SECTION D

10. State and prove Basic Proportionality Theorem. **4**