CHAPTER

1 Real Numbers

KEY POINTS

Decimal form of Real Numbers

Real Number (Q)

Irrational Number (I)

Terminating decimal (3/5, 5/4, …) denominator = $2^n 5^m$, where, $n\ m \in$ +ve integers

Non-terminating but, repeating decimal (2/3, 1/7, …)

Non terminating & non repating (1.010010001.., 1.232232223)

PROPERTIES OF REAL NUMBERS

Euclid division Lemma
given +ve integers a & b, there exist unique integers q & r satisfying $a = bq + r$, $0 \leq r < b$.

Fundamental Theorem of Airthmatic
Every composite number can be expressed factorised as a product of primes and this factorisation is unique apart from the order in which the prime factors occur.

Euclid division algorithm

The HCF of any two +ve integers a & b with $a > b$, is obtained as follows:

Step 1 : Apply Euclid division lemma to a & b to find q & r, where $a = bq + r$, $0 \leq r < b$.

Step 2 : If r = 0, the HCF is b If r ¹ 0 then apply Euclid 's lemma to b & r to find b_1, and r_1, where $b = b_1\ r + r_1$

Step 3 : If $r_1 = 0$, HCF is b, If r_1 ¹ 0, then continue the process. till $r_n = 0$ then bn at this stage will be HCF.

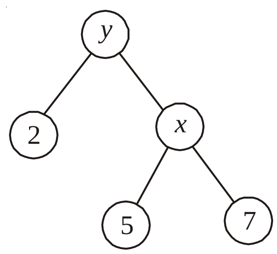
VERY SHORT ANSWER TYPE QUESTIONS

1. A number N when divided by 16 gives the remainder 5 ______ is the remainder when the same number is divided by 8.

2. HCF of $3^3 \times 5^4$ and $3^4 \times 5^2$ is ________ .

3. If $a = xy^2$ and $b = x^3y^5$ where x and y are prime numbers then LCM of (a, b) is _____ .

4. In factor tree find x and y



5. If n is a natural number, then $25^{2n} - 9^{2n}$ is always divisible by :

(i) 16 (ii) 34

(iii) both 16 or 34 (iv) None of these

6. The decimal expansion of the rational number $\dfrac{327}{2^3 \times 5}$ will terminate after

(a) One decimal place (b) Two decimal place

(c) Three decimal place (d) More than three decimal place

7. Which of the following rational numbers have terminating decimal?

(i) $\dfrac{16}{225}$ (ii) $\dfrac{5}{18}$ (iii) $\dfrac{2}{21}$ (iv) $\dfrac{7}{250}$

(a) (i) and (iii) (b) (ii) and (iii)

(c) (i) and (iii) (d) (i) and (iv)

8. Euclid's division Lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy.

(a) $1 < r < b$ (b) $0 < r \leq b$

(c) $0 \leq r < b$ (d) $0 < r < b$

9. $p^n = (a \times 5)^n$ For p^n to end with the digit zero $a =$ _____ for natural number n.

(a) any natural number (b) even number

(b) odd number (d) none of these

10. HCF is always

(a) multiple of LCM (b) Factor of LCM

(c) divisible by LCM (d) a and c both

11. All decimal numbers are

(a) rational number (b) irrational numbers

(c) real numbers (d) integers

12. Which of these numbers always end with the digits 6.

(a) 4^n (b) 2^n (c) 6^n (d) 8^n

13. Write the general form of an even integer

14. Write the form in which every odd integer can be written taking t as variable.

15. What would be the value of n for which $n^2 – 1$ is divisible by 8.

16. What can you say about the product of a non-zero rational and irrational number?

17. After how many places the decimal expansion of $\frac{13497}{1250}$ will terminate?

18. Find the least number which is divisible by all numbers from 1 to 10 (both inclusive).

19. The numbers 525 and 3000 are divisible by 3, 5, 15, 25 and 75 what is the HCF of 525 and 3000?

20. What will be the digit at unit's place of 9^n?

SHORT ANSWER TYPE QUESTIONS-I

21. If n is an odd integer then show that $n^2 – 1$ is divisible by 8.

22. Use Euclid's division algorithm to find the HCF of 16 and 28.

23. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

(NCERT Exemplar)

24. Without actual performing the long division, find if $\frac{395}{10500}$ will have terminating or non terminating (repeating decimal expansion.)

25. A rational number in its decimal expansion is 327. 7081. What can you say about the prime factors of q, when this number is expressed in the form of $\frac{p}{q}$? Give reasons.

26. What is the smallest number by which $\sqrt{5} - \sqrt{2}$ is to be multiplied to make it a rational number? Also find the number so obtained?

27. Find one rational and one irrational no between $\sqrt{3}$ and $\sqrt{5}$.

28. If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of m. (CBSE 2014)

29. Find the value of : $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$, where n is any positive and integer. (CBSE : 2016)

30. Show that any positive add integer is of the form 4q + 1 or 4q + 3, where q is some integer. (CBSE : 2012)

31. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times. (CBSE : 2016)

SHORT ANSWER TYPE QUESTIONS-II

32. Show that the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m.

33. Prove that $\sqrt{3}$ is an irrational number.

34. State fundamental theorem of Arithmetic and hence find the unique factorization of 120.

35. Prove that $\sqrt{3} + \sqrt{5}$ is irrational

36. Prove that $5 - \frac{3}{7}\sqrt{3}$ is an irrational number.

37. Prove that $\frac{1}{2-\sqrt{5}}$ is an irrational number.

38. Find HCF and LCM of 56 and 112 by prime factorization method.

39. Explain why:

(i) 7 × 11 × 13 × 15 + 15 is a composite number

(ii) 11 × 13 × 17 + 17 is a composite number.

(iii) 1 × 2 × 3 × 5 × 7 + 3 × 7 is a composite number.

40. On a morning walk, three perosns steps off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps? (NCERT Exemplar)

41. During a sale, colour pencils were being sold in the pack of 24 each and crayons in the pack of 32 each. If you want full packs of both and the same number of pencils and crayons, how many packets of each would you need to buy? (CBSE : 2017)

42. Find the largest number that divides 31 and 99 leaving remainder 5 and 8 respectively.

43. The HCF of 65 and 117 is expressible in the form 65 m – 117. Find the value of m. Also find the LCM of 65 and 117 using prime factorisation method.

44. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainder 1, 2 and 3 respectively. (NCERT Exemplar)

45. Show that square of any odd integer is of the form 4m + 1, for some integer m.

46. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm.

47. Find the greatest number of six digits exactly divisible by 18, 24 and 36.

48. Three bells ring at intervals of 9, 12, 15 minutes respectively. If they start ringing together at a time, after what time will they next ring together?

49. Show t hat only one of the number of n, $n + 2$ and $n + 4$ is divisble by 3.

50. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of two given number. (CBSE : 2018)

LONG ANSWER TYPE QUESTIONS

51. Find the HCF of 56, 96, 324 by Euclid's algorithm.

52. Show that any positive odd integer is of the form $6q + 1$, $6q + 3$ or $6q + 5$, where q is some integer.

53. Prove that the square of any positive integer is of the form $5q$, $5q + 1$, $5q + 4$ for some integer, q.

54. Prove that the product of three consecutive positive integers is divisible by 6.

55. For any positive integer n, prove that $n^3–n$ is divisible by 6. (NCERT Exemplar)

56. Show that one and only one of n, $n + 2$, $n + 4$ is divisible by 3.

57. Aakriti decided to distribute milk in an orphanage on her birthday. The supplier brought two milk containers which contain 398 *l* and 436 *l* of milk. The milk is to be transferred to another containers so that 7 *l* and 11 *l* of milk is left in both the containers respectively. What will be the maximum capacity of the drum?

58. Find the smallest number, which when increased by 17, is exactly divisible by both 520 and 468.

59. A street shopkeeper prepares 396 Gulab jamuns and 342 ras-gullas. He packs them, in combination. Each containter consists of either gulab jamuns or ras-gullab but have equal number of pieces.

Find the number of pieces he should put in each box so that number of boxes are least. (CBSE 2016)

60. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for integer q.

61. Express the HCF of numbers 72 and 124 as a linear combination of 72 and 124.

62. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

63. Find the HCF of numbers 134791, 6341 and 6339 by Euclid's div ision algorithm.

64. In a seminar, the no. of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the some the same number of participants are to be seated and all of the them being of the the same subject. (HOTS)

65. State fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

ANSWERS AND HINTS

1. 5

2. $3^3 \times 5^2$

3. $x^3 \times y^5$

4. $x = 35, y = 70$

5. (*iii*) $25^{2n} - 9^{2n}$ is of the form $a^{2n} - b^{2n}$ which is divisible by both $a - b$ and $a + b$ so, by both $25 + 9 = 34$ and $25 - 9 = 16$.

6. (c) three decimal place

7. (d) (i) and (iv)

8. (c) $0 \leq r < b$

9. (b) even number

10. (b) Factor of LCM

11. (c) real numbers

12. (c) 6^n

13. 2m

14. $2t + 1$

15. An odd integer

16. Irrational

17. 4

18. 2520

19. 75

20. 1 and 9

21. Any +ve odd integer is of the form 4q + 1 or 4q + 3 for some integer q so if $n = 4q + 1$.

$n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q = 8q(2q + 1) \Rightarrow n^2 - 1$ is divisible by 8.

If $n = 4q + 3$

$n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8\,(2q^2 + 3q + 1) \Rightarrow n^2 - 1$ is divisible by 8.

22. 4

23. As 12 has factors 2, 2, 3 it doesnot has 5 as its factor so 12^n will never end with 0 or 5.

24. Non-terminating repeating.

25. Denominator is the multiple of 2's and 5's.

26. $\sqrt{5} + \sqrt{2}$, 3

28. By Euclid's division lemma

$$180 = 144 \times 1 + 36$$
$$144 = 36 \times 4 + 0$$

HCF of 180 and 144 is 36.

29. Given that n is a positive odd integer

$\Rightarrow$ $2n$ and $4n + 2$ are even positive integers and n and $2n + 1$ are odd positive integers.

$\therefore$ $(-1)^n = -1, (-1)^{2n} = +1, (-1)^{2n+1} = -1, (-1)^{2n+2} = +1$

$\therefore$ $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2} = -1 + 1 - 1 + 1 = 0$

30. By applying Euclid division algorithm to a and b such that a = 4q + r, where $b = 4$, Now $r = 0, 1, 2, 3$.

where, $r = 0$, $a = 4q$ which is even number.

where, $r = 1$, $a = 4q + 1$ an odd number.

where, $r = 2$, $a = 4q + 2 = 2\,(2q + 1)$, an even number.

where, $r = 3$, $a = 4q + 3$ an odd number.

31. HCF of 850 and 680 is $2 \times 5 \times 17 = 170$ litres.

32. Let n be any psoitve integer. Then it is of the form $4q$, $4q + 1$, $4q + 2$ and $4q + 3$.

When $n = 4q$, $n^3 = 64q^3 = 4\,(16q^3) = 4\,m$, where $m = 16q^3$

When $n = 4q + 1$, $n^3 = (4q + 1)^3 = 64q^3 + 48q^2 + 12q + 1$

$$= 4(16q^3 + 12q^2 + 3q) + 1 = 4\,m + 1.$$

where $m = 16q^3 + 12q^2 + 3q$

Similarly discuss for n = $4q + 2$ and $4q + 3$.

34. $2 \times 2 \times 2 \times 3 \times 5$

35. Prove that $\sqrt{3}$ and $\sqrt{5}$ is irrational number separately and sum of two irrational number is an irrational number.

36. 5 is rational no. and $\frac{3}{7}\sqrt{3}$ is an irrational number. Difference of a rational number and irrational number is an irrational number.

38. HCF : 56, LCM : 112

39. (1) $15 \times (7 \times 11 \times 13 + 1)$ as it has more than two factors so it is composite no.

40. LCM of 40, 42, 45 = 2520

Minimum distance each should walk 2520 cm.

41. LCM of 24 and 32 is 96

96 crayons or $\frac{96}{32}$ = 3 packs of crayons

96 pencils or $\frac{96}{24}$ = 4 packs of pencils.

42. Given number = 31 and 99

$31 - 5 = 26$ and $99 - 8 = 91$

Prime factors of $26 = 2 \times 13$

$91 = 7 \times 13$

HCF of (26, 91) = 13.

$\therefore$ 13 is the largest number which divides 31 and 99 leaving remainder 5 and 8 respectively.

43. HCF of 117 and 65 by Euclid division algorithm.

$117 = 65 \times 1 + 52$

$65 = 52 \times 1 = 52$

$52 = 13 \times 4 + 0$

HCF (117, 52) = 13.

Given that $65\text{ m} - 117 = 13 \Rightarrow 65\text{ m} = 130 \Rightarrow m = 2$.

LCM (65, 117) = $13 \times 3^2 \times 5 = 585$

44. 1251 – 1 =1250, 9377 – 2 = 9375, 15628 – 3 = 15625

HCF of (15625, 9375) = 3125

HCF of (3125, 1250) = 625

$\Rightarrow$ HCF of (1250, 9375, 15625) = 625

45. By Euclid's division algorithm, we have $a = bq + r$, where $0 \le r < 4$. On putting $b = 4$ we get a = $4q + r$ where, $r = 0, 1, 2, 3$.

If $r = 0$, $a = 4q$ which is even

If $r = 1$, $a = 4q + 1$ not divisible by 2

If $r = 2$, $a = 4q + 2 = 2(2q + 1)$ which is even

If $r = 3$, $a = 4q + 3$ not divisible by 2.

So, for any +ve integer q, $4q + 1$ and $4q + 3$ are odd integers.

How, $a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 4(4q^2 + 2q) + 1 = 4m + 1$

where $m = 4q^2 + 2q$ similarly for $4q + 3$.

46. HCF (324, 252, 180) = 36

47. LCM of (18, 24, 36) = 72.

Greatest six digit number = 999999

```
72 ) 9 9 9 9 9 9 ( 1 3 8 8 8
   – 7 2
     2 7 9
   – 2 1 6
       6 3 9
     – 5 7 6
         6 3 9
       – 5 7 6
           6 3 9
           5 7 6
             6 3
```

Require six digit number

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  9 9 9 9 9 9
      –   6 3
  9 9 9 9 3 6
```

48. LCM of (9, 12, 15) = 180 minutes.

49. Let the number divisible by 3 is of the form $3k + r$, $r = 0, 1, 2$

a = 3k, $3k + 1$ or $3k + 2$

(*i*) When $a = 3k$

$n = 3k \Rightarrow n$ is divisible by 3.

$n + 2 = 3k + 2 \Rightarrow n + 2$ is not divisible by 3.

$n + 4 = 3k + 4 = 3k + 3 + 1 = 3(k + 1) + 1 \Rightarrow n + 4$ is not divisible by 3.

So, only one out of n, $n+2$ and $n+4$ is divisible by 3.

(*ii*) When $a = 3k + 1$

$n = 3k+1 \Rightarrow n$ is not divisible by 3.

$n + 2 = 3k + 1 + 2 = 3k + 3 = 3(k + 1)$

$\Rightarrow$ $n+2$ is divisible by 3.

$n + 4 = 3k + 1 + 4 = 3k + 5 = 3(k + 1) + 2$

$\Rightarrow n+2$ is not divisible by 3.

So, only one out of n, $n+2$ and $n+4$ is divisible by 3.

Similarly do for $a = 3k + 2$.

50. HCF (404, 96) = 4

LCM (404, 96) = 9696

HCF × LCM = 38, 784

Also, 404 × 96 = 38,784

51. 4

52. Let a be +ve odd integer, divide it by 6 then q is the quotient and r is the remiander.

$\Rightarrow$ $a = 6q + r$ where $r = 0, 1, 2, 3, 4, 5$

If, $a = 6q + 0 = 2(3q)$ is an even integer so not possible

If, $a = 6q + 1$ is an odd integer

If, $a = 6q + 2 = 2(3q + 1)$ is an even integer so not possible

If, $a = 6q + 3$ is an odd integer

If, $a = 6q + 4 = 2(3q + 2)$ is an even integer so not possible

If, $a = 6q + 5$ is an odd integer.

54. Let the three consecutive integers be a, a+1, a + 2,

Case I : If a is even,

$\Rightarrow$ $a + 2$ is the also even

$a(a + 2)$ is divisible by 2

a(a + 2) $(a + 1)$ is also divisible by 2

Now a, a + 1, a + 2 are three consecutive numbers

$\Rightarrow$ $a\,(a + 1)\,(a + 2)$ is a multiple by 3

$\Rightarrow$ $a\,(a + 1)\,(a + 2)$ is divisible by 3

as it is divisible by 2 and 3 hence divisible by 6.

Case II : If a is odd

$\Rightarrow$ $a + 1$ is even

$\Rightarrow$ $a + 1$ is divisible by 2

$\Rightarrow$ $a(a + 1)(a + 2)$ is also divisible by 2

Again a, $a + 1$, a + 2 are three consecutive numbers

$\Rightarrow$ $a(a + 1)(a + 2)$ is a multiple by 3

$\Rightarrow$ $a(a + 1)(a + 2)$ is divisible by 3

as it is divisible by 2 and 3 hence divisible by 6.

55.
$$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$$
$$= (n - 1)(n)(n + 1)$$
$$= \text{Product of three consecutive +ve integers}$$

Now to show that produce of three consecutive +ve integers is divisible by 6.

Any +ve integer a is of the form $3q$, $3q + 1$ or $3q + 2$ for some integer q.

Let a, $a + 1$, $a + 2$ be any three consecutive integers.

Case I : $a = 3q$

$(3q)(3q + 1)(3q + 2) = 3q(2m)$ [as $(3q + 1)$ and $(3q + 2)$ are consecutive integers so their product is also even]

$= 6q\text{m}$

which is divisible by 6.

Case II : If $a = 3q + 1$

$$a(a + 1)(a + 2) = (3q + 1)(3q + 2)(3q + 3)$$
$$= 2m^3(q + 1) \qquad (\text{as } (3q + 1)(3q + 2) = 2\text{m})$$
$$= 6\text{ m}(q + 1)$$

which is divisible by 6.

Case III : If $a = 3q + 2$

$$a(a + 1)(a + 2) = (3q + 2)(3q + 3)(3q + 4)$$
$$= (3q + 2)3(q + 1)(3q + 4)$$
$$= 6\text{ m}$$

which is divisible by 6.

57. 17

58. 4663

59. HCF (396, 342) = 18

61. HCF (124, 72) = 4

$$4 = 124 \times 7 + 72 \times (-12),\ x = 7,\ y = -12$$

62. Let $\sqrt{n-1} + \sqrt{n+1} = \dfrac{p}{q}$ (1)$q \neq 0$, p, q, co-prime.

$$\frac{q}{p} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}} \times \frac{\sqrt{n-1}-\sqrt{n+1}}{\sqrt{n-1}-\sqrt{n+1}}$$

$$\frac{q}{p} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$$

$$\sqrt{n-1}+\sqrt{n+1} = -\frac{2q}{p} \quad \text{or} \quad \sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \qquad \ldots(2)$$

Adding (1) & (2) we get $2\sqrt{n+1} = \dfrac{p}{q} + \dfrac{2q}{p} = \dfrac{p^2+2q^2}{pq}$ …(3)

Subtracting (1) & (2) we get $2\sqrt{n-1} = \dfrac{p^2-2q^2}{pq}$ …(4)

From (3) & (4) we get $\sqrt{n+1} + \sqrt{n-1}$ are rational numbers.

But $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number.

∴ These exist no positive integer n, for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

63. HCF (134791, 6341, 6339) = 1.

64. HCF of 60, 84 and 108 is $2^2 \times 3 = 12$

$$\text{No. of rooms required} = \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12} = 21 \text{ rooms}$$

65. HCF = 24, LCM = 540

$$\frac{\text{LCM}}{\text{HCF}} = \frac{540}{24} = 22.5, \text{ not an integer.}$$

Hence two numbers cannot have HCF and LCM as 24 and 540 respectively.

PRACTICE-TEST

Real Number

Time : 1 Hr. ***M.M. : 20***

SECTIONA

1. After how many decimal places the decimal expansion of $\frac{51}{150}$ will terminate. 1

2. In Euclid's Division Lemma, when $a = bq + r$ where a, b are positive integers then what values r can take? 1

3. HCF of x^4y^5 and x^8y^3. 1

4. LCM of 14 and 122 . 1

SECTION B

5. Show that 9^n can never ends with unit digit zero. 2

6. Without actual division find the type of decimal expansion of $\frac{935}{10500}$ 2

7. Show that the square of any odd integer is of the form $4m + 1$, for some integer m. 2

SECTION C

8. Prove that $\frac{1}{3-2\sqrt{5}}$ is an irrational number. 3

9. Find the HCF of 36, 96 and 120 by Euclid's Lemma. 3

SECTION D

10. Once a sports goods retailer organized a campaign "Run to remember" to spread awareness about benefits of walking. In that Soham and Baani participated. There was a circular path around a sports field. Soham took 12 minutes to drive one round of the field, while Baani took 18 minutes for the same. Suppose they started at the same point and at the same time and went in the same direction.

After how many minutes have they met again at the starting point? 4

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