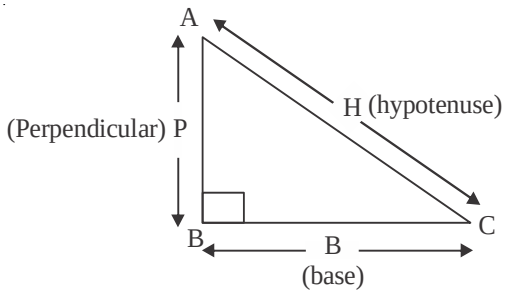
CHAPTER

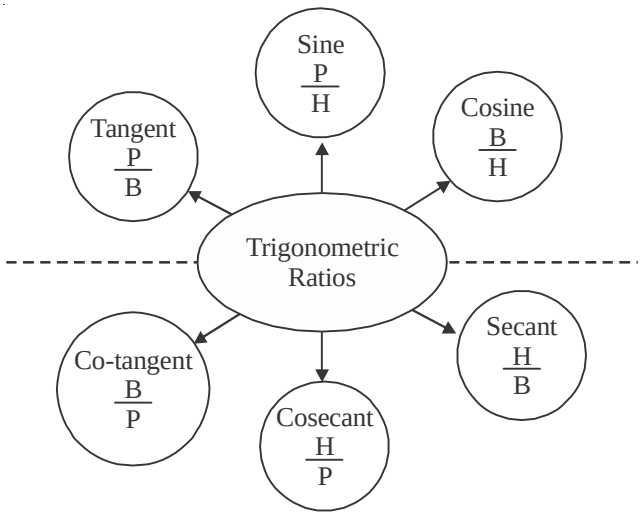
8 Introduction to Trigonometry

KEY POINTS



- A branch of mathematics which deals with the problems related to right angled triangles. It is the study of relationship between the sides and angles of a right angled triangel.
  Note : For ∠A — Perpendicular is BC base is AB.
  For ∠C, Perpendicualr is AB Base is BC.

Trigonometric Rations of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.

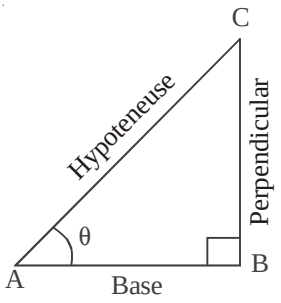


Mind Trick: To learn the relationship of sine, cosine and tangent follow this sentences.

Some People Have Curly Brown Hair Through Proper Brushing

$\sin A = \frac{P}{H}$ $\cos A = \frac{B}{H}$ $\tan A = \frac{P}{B}$

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Opposites

$$\sin\theta = \frac{1}{\text{cosec }\theta}, \ \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \ \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \ \cot\theta = \frac{1}{\tan\theta}$$

3. $\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

4. Identities

$\sin^2\theta + \cos^2 = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. Trigonometric ratios of some specific angles

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. Trigonometric ratios of complimentary angles

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \text{Sin}\,\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \text{cosec}\,\theta$

$\text{cosec}(90° - \theta) = \sec\theta$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\text{Sin}\,\theta = \cos\theta$, find the value of θ

2. If $\tan\theta = \cot(30° + \theta)$, find the value of θ

3. If $\text{Sin}\,\theta = \cos(\theta - 6°)$, find the value of θ

4. If $\cos A = \frac{7}{25}$, find the value of $\tan A + \cot A$

5. If $\tan\theta = \frac{4}{3}$ then find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

6. If $3x = \text{cosec}\ \theta$ and $\frac{3}{x} = \cot\theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$

7. If $x = a \sin\theta$ and $y = a \cos\theta$ then find the value of $x^2 + y^2$

8. Find the value of $\text{cosec}\ 70° - \sec 20°$

9. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$ then find the value of $5\left(x^2 - \frac{1}{x^2}\right)$

10. Find the value of $9\sec^2 A - 9\tan^2 A$

11. Express $\sec\theta$ in terms of $\cot\theta$

12. Find the value of $\cos\theta \cos(90° - \theta) - \sin\theta \sin(90° - \theta)$

13. If $\sin(20° + \theta) = \cos 30°$ then find the value of θ.

14. Find the value of $\frac{1+\tan^2\theta}{1+\cot^2\theta}$

15. Find the value of $\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

16. Given $\tan\theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$. (CBSE, 2010)

17. If $\theta = 45°$, then find the vlaue of $\text{cosec}^2\theta$. (CBSE, 2010)

18. If $\cos\theta = \frac{2}{3}$, then find the value of $2\sec^2\theta + 2\tan^2\theta - 7$. (CBSE, 2011)

19. Find the value of $6\tan^2\theta - 6\sec^2\theta$

20. Express $\text{cosec}\ 48° + \tan 88°$ in terms of trigonometric ratios of angle between $0°$ and $45°$.

21. If $5\tan\theta - 4 = 0$, then value of $\frac{5\sin\theta - 4\sin\theta}{5\sin\theta + 4\cos\theta}$ is

(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{6}$

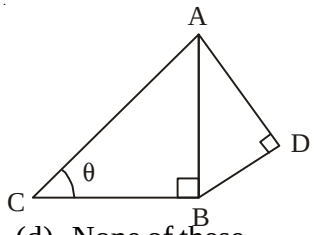
22. If A and B are complementary angles, then

(a) $\sin A = \sin B$ (b) $\cos A = \cos B$ (c) $\tan A = \tan B$ (d) $\sec A = \operatorname{cosec} B$

23. In Fig. if $AD = 4$ cm, BD = 3 cm and CB = 12 cm. then $\cot \theta =$

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$

(c) $\frac{13}{12}$ (d) $\frac{12}{13}$



24. The value of tan 1°, tan 2°, tan 3° _____ tan 89° is.

(a) 1 (b) – 1 (c) 0 (d) None of these

25. If θ and $2\theta - 45°$ are acute angles such that $\sin \theta = \cos (2\theta - 45°)$ then $\tan \theta$ is

(a) 1 (b) – 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

SHORT ANSWER TYPE (I) QUESTIONS

Prove that :

26. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

27. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan \theta + \text{Sec } \theta$

28. If $x = p \sec \theta + q \tan \theta$ & $y = p \tan \theta + q \sec \theta$ then prove that $x^2 - y^2 = p^2 - q^2$

29. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ then show that $\tan \theta = \frac{1}{\sqrt{3}}$

30. If $\text{Sin } (A - B) = \frac{1}{2}$, $\cos (A + B) = \frac{1}{2}$ then find the value of A and B.

31. Find the value of $\frac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$.

32. **Prove that :** $\tan 1° \tan 11° \tan 21° \tan 69° \tan 79° \tan 89° = 1$

33. If $\sec 4A = \operatorname{cosec} (A - 20°)$ then find the value of A.

34. If 3 cot A = 4, find the value of $\frac{\text{Cosec}^2 A + 1}{\text{Cosec}^2 A - 1}$.

35. If tan (3x – 15°) = 1 then find the value of x.

36. If A, B, C are interior angles of ΔABC, the prove that $\cos ec\left(\frac{A+B}{2}\right) = \sec\left(\frac{C}{2}\right)$.

(CBSE 2011)

37. In ΔABC, right angled at B, AB = 5 cm and ∠ACB = 30°. Find BC and AC.

38. If tan θ = cot (30° + θ), Find the value of θ. (CBSE, 2012)

39. Show that : $\frac{1 - \sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

40. Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 4$, $\theta \le 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS

Prove that :

41. $\frac{\tan A + \text{Sec}\, A - 1}{\tan A - \text{Sec}\, A + 1} = \frac{1 + \text{Sin}\, A}{\text{Cos}\, A}$

42. $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

43. $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta = \sec\theta \,\text{cosec}\,\theta + 1$

44. $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

45. sec A (1 – sin A) (sec A + tan A) = 1

46. If tan θ + sin θ = m, tan θ – sin θ = n then show that $m^2 - n^2 = 4\sqrt{mn}$.

47. If sec θ = $x + \frac{1}{4x}$, prove that sec θ + tan θ = 2x or $\frac{1}{2x}$

48. If sin θ + $\sin^2\theta$ = 1, prove that $\cos^2\theta + \cos^4\theta = 1$

49. Without using trigonometric table, the value of
cot θ tan (90° – θ) – sec (90° – θ) cosec θ + $\sin^2 65°$ + $\sin^2 25°$ + $\sqrt{3}$ tan 5° tan 85°.

50. Prove that : $\frac{\cot(90°-\theta)}{\tan\theta}+\frac{\operatorname{cosec}(90°-\theta)\sin\theta}{\tan(90°-\theta)}=\sec^2\theta$

51. Find the value of :

$\frac{\cos 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40} + 2\ \text{Cosec}^2\ 58° - 2\ \text{Cot}\ 58°\ \tan 32° - 4\ \tan 13°\ \tan 37°\ \tan 77°\ \tan 45°\ \tan 53°.$

52. If A, B, C are the angles of ΔABC then prove that $\operatorname{cosec}^2\left(\frac{B+C}{2}\right)-\tan^2\frac{A}{2}=1$

53. Find the value of $\sec^2 10° - \cot^2 80° + \frac{\sin 15°\cos 75° + \cos 15°\sin 75°}{\cos\theta\sin(90°-\theta)+\sin\theta\cos(90°-\theta)}$.

54. Prove that : $\frac{\tan\theta-\cot\theta}{\sin\theta\cos\theta}=\tan^2\theta-\cot^2\theta$. (CBSE 2012)

55. If $\cos\theta+\sin\theta=\sqrt{2}\cos q$, then show that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$.

56. Evaulate : $4-\frac{\sin 30°+\tan 45°-\operatorname{cosec} 60°}{\sec 30°+\cos 60°+\cos 45°}$ (CBSE, 2012)

57. Prove that : $1-\frac{\sin A\sin(90°-A)}{\cot(90°-A)}=\sin^2 A$ (CBSE, 2012)

58. If $a\cos\theta = b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$ (CBSE, 2001 C)
Prove that : $a^2 + b^2 = m^2 + n^2$

59. If $a\cos\theta - b\sin\theta = c$ prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$.

(CBSE, 2001 C)

60. Without using trigonometric tablets, evaluate :

$\frac{\sec^2 54° - \cot^2 36°}{\operatorname{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38°\sec^2 52° - \sin^2 45°$ (CBSE, 2005)

LONG ANSWER TYPE QUESTIONS

Prove That:

61. $\frac{\text{Sec}\,\theta + \tan\theta - 1}{\tan\theta - \text{Sec}\,\theta + 1} = \frac{\text{Cos}\,\theta}{1 - \text{Sin}\,\theta}$

62. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\text{Cot}^2\theta}\right) = \frac{1}{\text{Sin}^2\theta - \text{Sin}^4\theta}$

63. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

64. $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A \cos A$

65. If $\text{Sin}\,\theta + \text{Cos}\,\theta = m$ and $\text{Sec}\,\theta + \text{Cosec}\,\theta = n$ then show that $n(m^2 - 1) = 2m$

66. find the value of :

$$\frac{\text{Cot}(90° - \theta)\tan\theta - \text{Cosec}(90° - \theta)\text{Sec}\,\theta}{\text{Sin}\,12°\,\text{Cos}\,15°\,\text{Sec}\,78°\,\text{Cosec}75°} + \frac{\text{Cos}^2(50° + \theta)\tan^2(40° - \theta)}{\tan 15°\tan 37°\tan 53°\tan 75°}$$

67. Prove that :

$$\frac{1}{\text{Cosec}\,\theta + \text{Cot}\,\theta} - \frac{1}{\text{Sin}\,\theta} = \frac{1}{\text{Sin}\,\theta} - \frac{1}{\text{Cosec}\,\theta - \text{Cot}\,\theta}$$

68. If $\frac{\text{Cos}\,\alpha}{\text{Cos}\,\beta} = m$ and $\frac{\text{Cos}\,\alpha}{\text{Sin}\,\beta} = n$, then prove that $(m^2 + n^2)\,\text{Cos}^2\beta = n^2$

69. If $\tan\theta + \text{Sin}\,\theta = m$. $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$

70. Prove that :

$$\text{Sec}^2\theta - \frac{\text{Sin}^2\theta - 2\text{Sin}^4\theta}{2\text{Cos}^4\theta - \text{Cos}^2\theta} = 1$$

71. $\text{Cot}\,\theta\tan(90° - \theta) - \text{Sec}(90° - \theta)\,\text{Cosec}\,\theta + \sqrt{3}\tan 12°\tan 60°\tan 78°$ find its value.

72. Find the value of —

$$\frac{\text{Sec}(90° - \theta)\,\text{Cosec}\,\theta - \tan(90° - \theta)\,\text{Cot}\,\theta + \text{Cos}^2 25° + \text{Cos}^2 65°}{3\tan 27°\tan 63°}$$

ANSWERS AND HINTS

1. 45°

2. 30°

3. 24°

4. $\frac{625}{168}$

5. 7 **6.** $\frac{1}{3}$

7. a **8.** 0

9. $\frac{1}{5}$ **10.** 9

11. $\frac{\sqrt{1+\text{Cos}^2\,\theta}}{\text{Cot}\,\theta}$ **12.** 0°

13. 50° **14.** $\tan^2\theta$

15. $\tan\theta$ **16.** $\frac{1}{2}$

17. 2 **18.** 0

19. – 6 **20.** sec 42° + cot 2°

21. (c) **22.** (d)

23. (a) **24.** (a)

25. (a) **26.** —

27. — **28.** —

29. — **30.** A = 45°, B = 15°

31. 1 **32.** —

33. 22° **34.** $\frac{17}{8}$

35. 20° **36.** Hint : A + B + C = 180°

37. AC = 10 cm, BC = $5\sqrt{3}$ cm **38.** 30°

40. 60° **49.** $\sqrt{3}$

51. – 1 **53.** 2

56. $\frac{20+9\sqrt{3}}{4+3\sqrt{3}}$ **60.** $\frac{2+2\sqrt{3}}{2}$

71. 2 **72.** $\frac{2}{3}$

PRACTICE-TEST

Introduction to Trigonometry

Time : 1 Hrs. ***M.M.: 20***

SECTION-A

1. If $\text{Sin}\,\theta = \frac{4}{5}$ what is the value of $\cos\theta$. **1**

2. Write the value of $\text{Sin}\,(45° + \theta) - \text{Cos}\,(45° - \theta)$. **1**

3. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is **1**

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) 0

4. If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is **1**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

SECTION-B

5. If $5\tan\theta = 4$ then find the value of $\frac{5\,\text{Sin}\,\theta - 3\text{Cos}\,\theta}{5\,\text{Sin}\,\theta + 2\text{Cos}\,\theta}$ **2**

6. Find the value of $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$ **2**

7. Prove that $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \text{cosec}\,\alpha$ **2**

SECTION-C

8. Prove that $\frac{\text{Sin}\,\theta}{1 + \text{Cos}\,\theta} + \frac{1 + \text{Cos}\,\theta}{\text{Sin}\,\theta} = 2\,\text{Cosec}\,\theta$ **3**

9. Prove that $\frac{\text{Cos}\,A}{1 - \tan A} - \frac{\text{Sin}^2 A}{\text{Cos}\,A - \text{Sin}\,A} = \text{Sin}\,A + \text{Cos}\,A$ **3**

SECTION-D

10. Prove that $\frac{\tan\theta + \text{Sec}\,\theta - 1}{\tan\theta - \text{Sec}\,\theta + 1} = \frac{\text{Cos}\,\theta}{1 - \text{Sin}\,\theta}$. **4**

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