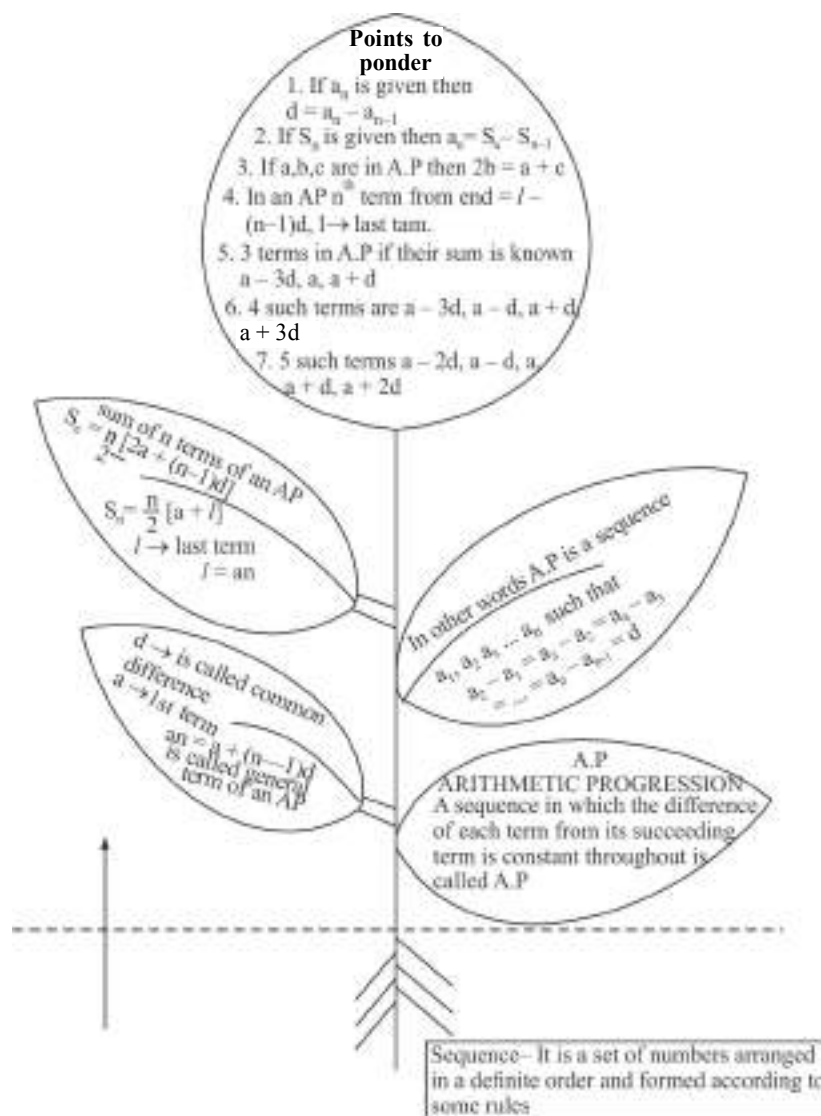


CHAPTER

5

Arithmetic Progression



VERY SHORT ANSWER TYPE QUESTIONS

- Find 5th term of an A.P. whose n^{th} term is $3n - 5$
- Find the sum of first 10 even numbers.
- Write the n^{th} term of odd numbers.
- Write the sum of first n natural numbers.
- Write the sum of first n even numbers.
- Find the n^{th} term of the A.P. $-10, -15, -2, -25, \dots$
- Find the common difference of A.P. $4\frac{1}{9}, 4\frac{2}{9}, 4\frac{1}{3}, \dots$
- Write the common difference of an A.P. whose n^{th} term is $a_n = 3n + 7$
- What will be the value of $a_8 - a_4$ for the following A.P.
 $4, 9, 14, \dots, 254$
- What is value of a_{16} for the A.P. $-10, -12, -14, -16, \dots$
- $3, k - 2, 5$ are in A.P. find k .
- For what value of p , the following terms are three consecutive terms of an A.P. $\frac{4}{5}, p, 2$.
- In the following A.Ps, find the missing terms in the boxes : (NCERT)**
 - $2, \square, 26$ (b) $\square, 13, \square, 3$
 - $5, \square, \square, 9\frac{1}{2}$ (d) $-4, \square, \square, \square, \square, 6$
 - $\square, 38, \square, \square, \square, -22$
- Multiple Choice Questions:**
 - 30th term of the A.P. $10, 7, 4 \dots$ is
 - 97
 - 77
 - 77
 - 87
 - 11th term of an A.P. $-3, -\frac{1}{2}, \dots$ is
 - 28
 - 22
 - 38
 - $-48\frac{1}{2}$

- (c) In an A.P. if $d = -4$, $n = 7$, $a_n = 4$, then a is
 (A) 6 (B) 7
 (C) 120 (D) 28
- (d) The first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$ then y equals: **(CBSE 2014)**
 (A) -3 (B) 4
 (C) 5 (D) 2
- (e) The list of numbers $-10, -6, -2, 2, \dots$ is
 (A) An A.P. with $d = -16$ (B) An A.P. with $d = 4$
 (C) An A.P. with $d = -4$ (D) Not an A.P.
- (f) The 11th term from the last term of an A.P. $10, 7, 4, \dots, -62$ is **(NCERT)**
 (A) 25 (B) -32
 (C) 16 (D) 0
- (g) The famous mathematician associated with finding the sum of the first 100 natural numbers is
 (A) Pythagoras (B) Newton
 (C) Gauss (D) Euclid
- (h) What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?
 (A) 8 (B) -8
 (C) -4 (D) 4

15. Match the following :

Column A

Column B

- | | |
|---|---------------------|
| (a) $a = -18$, $n = 10$, $d = 2$ then an of A.P. | (a) $\frac{a+c}{2}$ |
| (b) a , b and c in A.P. then their Arithmetic mean is | (b) 0 |
| (c) If 2, 4, 6, are in A.P. then 4, 8, 12 will also be an | (c) -41 |
| (d) If $a_n = 9 - 5n$ of an A.P. then a_{10} will be | (d) 8 |
| (e) If $d = -2$, $n = 5$ and $a_n = 0$ in A.P. then a is | (e) A.P. |

16. State True/False and justify

- (a) 301 is a term of A.P. 5, 11, 17, 23 **(NCERT)**
- (b) Difference of m th and n th term of an A.P. $= (m - n) d$.
- (c) 2, 5, 9, 14, is an A.P.
- (d) Sum of first 20 natural numbers is 410.
- (e) n th term of A.P. 5, 10, 15, 20 n terms and n th term of A.P. 15, 30, 45, 60, ... n terms are same.

SHORT ANSWER TYPE QUESTIONS-I

17. Is 144 a term of the A.P. 3, 7, 11, ? Justify your answer.
18. Find the 20th term from the last term of the A.P. 3, 8, 13,, 253
19. Which term of the A.P. 5, 15, 25, will be 130 more than its 31st term?
20. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively, Find the sum of all terms of this A.P.
21. Find the sum of first 15 multiples of 8.
22. Is the sequence formed in the following situations an A.P.
 - (i) Number of students left in the school auditorium from the total strength of 1000 students when they leave the auditorium in batches of 25.
 - (ii) The amount of money in the account every year when Rs. 100 are deposit annually to accumulate at compound interest at 4% per annum.
23. Find the sum of even positive integers between 1 and 200.
24. If $4m + 8$, $2m^2 + 3m + 6$, $3m^2 + 4m + 4$ are three consecutive terms of an A.P. find m .
25. How many terms of the A.P. 22, 20, 18, should be taken so that their sum is zero.
26. If 10 times of 10th term is equal to 20 times of 20th term of an A.P. Find its 30th term.
27. Find the middle term of the A.P. 6, 13, 2, 216.
28. Find whether (-150) is a term of A.P. 11, 8, 5, 2, ? **(NCERT)**
29. Find how many two digit numbers are divisible by 6? **(CBSE 2011)**
30. If $\frac{1}{x+2}$, $\frac{1}{x+3}$ and $\frac{1}{x+5}$ are in A.P. find x . **(CBSE 2011)**
31. Find the middle term of an A.P. $-6, -2, 2, \dots 58$. **(CBSE 2011)**
32. In an A.P. find S_n , where $a_n = 5n - 1$. Hence find the sum of the first 20 terms. **(CBSE 2011)**
33. Which term of A.P. 3, 7, 11, 15 is 79? Also find the sum $3 + 7 + 11 + \dots + 79$. **(CBSE 2011C)**
34. Which term of the A.P. : 121, 117, 113 ... is the first negative terms ? **(NCERT)**
35. Find the 20th term from the last term of the A.P. 3, 8, 13, ... 253. **(NCERT)**

SHORT ANSWER TYPE QUESTIONS-II

36. Find the middle terms of the A.P. 7, 13, 19,, 241.
37. Find the sum of integers between 10 and 500 which are divisible by 7.
38. The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th term is 97. Find the A.P.
39. If the m th term of an A.P. be $\frac{1}{n}$ and n th term be $\frac{1}{m}$, show that its (mn) th is 1.
40. If the p th of term A.P. is q and the q th term is p , prove that its n th term is $(p + q - n)$.
41. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.
42. The sum of 5th and 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, find the A.P.
43. If S_n , the sum of first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, then find its n th term and common difference.
44. Which term of the A.P. 3, 15, 27, 39 will be 120 more than its 21st term?
(CBSE 2018)
45. If S_n , the sum of first n terms of an A.P. is given by $S_n = 3x^2 - 4x$, find the n th term.
(CBSE 2018)
46. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?
(NCERT)
47. For what value of n , are the n th term of two A.P's 63, 65, 67 and 3, 10, 17 are equal ?
(NCERT)
48. Which term of an A.P. 3, 15, 27, 39 will be 132 more than its 54th term?
(NCERT)
49. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, find the 20th term.
(NCERT)
50. Find the sum of odd numbers between 0 and 50.
(NCERT)
51. If $S_n = 4n - n^2$ in an A.P. find the A.P.
(NCERT)
52. How many terms of the A.P. 9, 17, 25, must be taken to give a sum of 636?
(NCERT)

LONG ANSWER TYPE QUESTIONS

53. The sum of third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first 16th terms of the A.P.
54. Determine the A.P. whose 4th term is 18 and the difference of 9th term from the 15th term is 30.
55. The sum of first 9 terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1:2. Find the first and fifteenth terms of the A.P.
56. If the 10th term of an A.P. is 21 and the sum of its first 10 terms is 120, find its n^{th} term.
57. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P.
58. The sum of first 20 terms of an A.P. is one third of the sum of next 20 terms. If first term is 1, find the sum of first 30 terms of this A.P.
59. If the sum of the first four terms of an AP is 40 and the sum of the first fourteen terms of an AP is 280. Find the sum of first n terms of the A.P. (CBSE 2018)
60. Ramkali required Rs. 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly savings by ₹ 20 every week. Find whether she will be able to send her daughter to school after 12 weeks. (CBSE 2015)
61. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of last 15 terms is 2565. Find the A.P. (CBSE 2014)
62. The sum of first n terms of an A.P. is $5n^2 + 3n$. If the m^{th} term is 168, find the value of m . Also find the 20th term of the A.P. (CBSE 2013)
63. If the sum of the first seven terms of an A.P. is 49 and the sum of its first 17 terms is 289. Find the sum of first n terms of an A.P. (CBSE 2016)
64. If the 4th term of an A.P. is zero, prove that the 25th term of the A.P. is three times its 11th term. (CBSE 2016)
65. In an A.P. if $S_5 + S_7 = 167$ and $S_{10} = 235$. Find the A.P., where S_n denotes the sum of its first n terms. (CBSE 2015)
66. In an AP prove $S_{12} = 3(S_8 - S_4)$ where S_n represent the sum of first n terms of an A.P. (CBSE 2015)

15. (a) \rightarrow (b) (b) \rightarrow (a)
 (c) \rightarrow (e) (d) \rightarrow (c)
 (e) \rightarrow (d)

16. (a) False, $301 = 5 + (n - 1) 6$

Solving we get $n = \frac{151}{3}$ which is not a natural number.

\therefore 301 is not a term of this A.P.

- (b) True $[a + (m - 1) d] - [a + (n - 1) d] = (m - n) d$

- (c) False $\because a_2 - a_1 = 5 - 2 = 3$

$$\because a_3 - a_2 = 9 - 5 = 4$$

- (d) False $\because S_n = \frac{n(n+1)}{2} = \frac{20 \times 21}{2} = 210$

- (e) True (If $a, b, c, d \dots$ are in AP then $ka, kb, kc, kd \dots$ are in AP)

$$k \neq 0$$

- (f) $144 = 3 + (n - 1) 4$

$$\frac{141}{4} + 1 = n \text{ which is not possible}$$

18. No, use $l - (n - 1) d$

Ans. 158

19. Let $a_n = 130 + a_{31}$
 Solve to get $n = 44$

Ans. 44th term

20. $a = 12, d = 6, a_n = 252 \Rightarrow n = 41$

Find $S_{41} = 5412$, use $S_n = \frac{n}{2} [2a + (n - 1) d]$

21. $S_{15} = \frac{15}{2} [2a + 14d]$

where $a = 8, d = 8$

Ans. 960

22. (i) Yes (ii) No

23. $2 + 4 + 6 + \dots + 198$

$$a = 2, d = 2, a_n = 198 \Rightarrow n = 99$$

$$S_n = \frac{n}{2} [a + l] = 9900$$

24. $b = \frac{a+c}{2}$

$$\therefore 2m^2 + 3m + 6 = \frac{4m+8+3m^2+4m+4}{2}$$

Solve to get $m^2 - 2m = 0$

$$m = 0, 2$$

25. $S_n = 0 \Rightarrow \frac{n}{2} [44 + (n-1)(-2)] = 0.$

Solve $n = 23$

26. ATQ $10 a_{10} = 20 a_{20}$

$$\Rightarrow a_{10} = 2a_{20}$$

$$a + 9d = 2a + 38d$$

$$a = -29d \dots(1)$$

$$a_{30} = a + 29d$$

Substitute a from (1)

Ans. $a_{30} = 0$

27. 6, 13, 20, ..., 216

Find n from $a_n = a + (n-1)d$

then use concept of median

Middle term = 111.

28. Let $a_n = -150$

$$11 + (n-1)(-3) = -150$$

Solve and get n is not a natural number.

\therefore **Ans.** No.

29. Two digit No.s divisible by 6 are 12, 18, 24, 96.

$$a_2 - a_1 = a_3 - a_2 = 6$$

$$\therefore \text{A.P., } a_n = 96 \Rightarrow n = 15$$

30. $\frac{2}{x+3} = \frac{1}{x+2} + \frac{1}{x+5}$ ($2b = a + c$)

Solve to get $x = 1.$

- 31.** $a_n = a + (n - 1) d$
 $58 = -6 + (n - 1) 4$
 find $n = 17$
 Find Middle term using concept of median

$$= \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} = 9\text{th term}$$

$$a_9 = -6 + 8(4) = 26$$

- 32.** $a_n = 5n - 1$
 Find AP $a_1 = 4, a_2 = 9, a_3 = 14$
 $4, 9, 14, \dots$

$$a_2 - a_1 = 5 = a_3 - a_2$$

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} [8 + (n - 1) 5]$$

$$= \frac{n}{2} [5n + 3]$$

$$S_{20} = \frac{20}{2} [100 + 3] = 10 \times 103 = 1030$$

- 33.** $79 = 3 + (n - 1) 4$
 $n = 26$

$$S_{26} = \frac{26}{2} [3 + 79] = 13[82]$$

$$S_{26} = 1066$$

- 34.** Let $a_n < 0$
 $121 + (n - 1) (-4) < 0$
 $121 - 4n + 4 < 0$
 $125 < 4n$

$$n > \frac{125}{4}$$

$$\therefore n = 32$$

32nd term will be first negative term.

- 35.** 20th term from end using $[l - (n - 1) d]$
 $= 253 - 19 \times 5$
 $= 253 - 95 = 158$

SHORT ANSWER TYPE QUESTIONS-II

36. Same as Q.27.

Ans. 121, 127

37. No.s between 10 and 500 which are divisible by 7, 14, 21, 28 ..., 497

Find n , using $a_n = a + (n - 1) d$, then use $S_n = \frac{n}{2} [2a + (n - 1) d]$

Ans. $S_n = 17885$.

38. $a_5 + a_9 = 72$

$$a_7 + a_{12} = 97$$

Solve these equations to get a and d .

A.P., 6, 11, 16, 21, 26,

$$39. \quad a_m = \frac{1}{n} \Rightarrow a + (m - 1)d = \frac{1}{n}$$

$$a_n = \frac{1}{m} \Rightarrow a + (n - 1)d = \frac{1}{m}$$

$$(m - n) d = \frac{1}{n} - \frac{1}{m} = \frac{m - n}{mn}$$

$$\therefore d = \frac{1}{mn}, \text{ find } a = \frac{1}{mn}$$

$$a_{mn} = a + (mn - 1) d$$

$$= \frac{1}{mn} + (mn - 1) \frac{1}{mn}$$

$$a_{mn} = 1.$$

40. $a_p = q, \quad a_q = p$

Solve to get a and d then find $a_{p+q-n} = 0$

41. No.s divisible by both 2 and 5

\Rightarrow No.s divisible by 10.

No.s between 101 and 999 divisible by 2 and 5 both 110, 120, 130, 140, ..., 990.

Use $a_n = 990$ to get $n = 89$.

42. ATQ $a_5 + a_9 = 30$
 $a_{25} = 3 a_8$
 Solve to get $a = 3, d = 2$
 A.P. 3, 5, 7, 9, ...
43. $S_n = 5n^2 + 3n$
 Find $a_n = S_n - S_{n-1} = 10n - 2$
 Use it to get $d = 10$
44. Let $a_n = 120 + a_{21}$
 $3 + (n - 1)d = 120 + [3 + 20d]$
 $3 + (n - 1)12 = 120 + [3 + 20 \times 12]$
 $= 120 + 243$
 $(n - 1)12 = 363 - 3 = 360$
 $n = 31$
45. $S_n = 3n^2 - 4n$
 $a_n = S_n - S_{n-1}$
 $= (3n^2 - 4n) - [3(n - 1)^2 - 4(n - 1)]$
 $= (3n^2 - 4n) - [3n^2 + 3 - 6n - 4n + 4]$
 $= - [7 - 6n]$
 $a_n = 6n - 7$
46. 23, 21, 19, ... 5
 $a_n = a + (n - 1) d$
 $S = 23 + (n - 1) (-2)$
 $n = 10$
47. 63, 65, 67,
 $a_n = 63 + (n - 1) 2$
 $= 61 + 2n$
 3, 10, 17,
 $a_n = 3 + (n - 1) 7$
 $= 7n - 4$
 $61 + 2n = 7n - 4$
 $65 = 5n$
 $n = 13$

48. 65th term

(NCERT)

49. $S_{14} = 1050, a = 10$

$$S_{14} = \frac{14}{2} [2 \times 10 + 13d]$$

$$\frac{1050}{7} = 20 + 13d$$

$$\frac{150 - 20}{13} = d \Rightarrow d = 10$$

$$a_{20} = a + 19d = 10 + 190 = 200$$

50. Odd no.s between 0 to 50

$$1, 3, 5, 7, \dots, 49$$

$$a_n = 49$$

$$a + (n - 1)d = 49$$

$$1 + (n - 1)2 = 49$$

$$n = 25$$

$$S_n = \frac{n}{2} [a + l]$$

$$S_{25} = \frac{25}{2} [1 + 49] = 25 \times 25 = 625$$

51. $S_n = 4n - n^2$

$$S_1 = a_1 = 4 - 1 = 3$$

$$S_2 = a_1 + a_2 \Rightarrow a_2 = 1 \quad \text{AP } 3, 1, -1, \dots$$

$$S_3 = a_1 + a_2 + a_3 = -1$$

52. $n = 12, n = -\frac{53}{4}$

(NCERT)

LONG ANSWER TYPE QUESTIONS

53. $a_3 + a_7 = 6$ $a = 1, d = \frac{1}{2}, S_n = 76$
will give $a = 5, d = -\frac{1}{2}, S_n = 20$
 $a_3 \times a_7 = 8$

Ans. 76, 20

54. ATQ $a_4 = 18 \dots(1), a_{15} - a_9 = 30 \dots(2)$

equation (2) will give $d = 5$

Substitute $d = 5$ in (1) to get $a = 3$

A.P. 3, 8, 13,

55. ATQ $S_9 = 162 \Rightarrow \frac{9}{2} [2a + 8d] = 162 \dots(1)$

ATQ $\frac{a_6}{a_{13}} = \frac{1}{2}$ solve and get $a = 2d$

Sub $a = 2d$ in (1) to get $d = 3, a = 6$

$$a_{15} = a + 14d$$

Ans. $a_{15} = 48$

56. $a_{10} = 21, S_{10} = 120$. Solve these to get a and d then find

$$a_n = a + (n - 1)d$$

Ans. $a_n = 2n + 1$

57. ATQ $S_7 = 63,$

$$\dots(1)$$

Sum of next 7 terms $= S_{14} - S_7 = 161$

$$\dots(2)$$

Use $S_n = \frac{n}{2} [2a + (n - 1)d]$

Solve (1) and (2) to get a and d then find a_{28} using $a_n = a + (n - 1)d$.

Ans. $a_{28} = 57$

58. ATQ $S_{20} = \frac{1}{3}(S_{40} - S_{20}), a = 1$

Use $S_n = \frac{n}{2} [2a + (n - 1)d]$ and $a = 1$ to find d

then find S_{30} .

Ans. 900

59. $S_4 = 40 \Rightarrow \frac{4}{2} [2a + 3d] = 40$

$$S_{14} = 280 \Rightarrow \frac{14}{2} [2a + 13d] = 280$$

Solve to get $a = 7, d = 2$

60. $a = 100, d = 20, n = 12$

$$S_{12} = \frac{12}{2} [200 + 220] = 6 \times 420$$

$$= 2520 > 2500$$

\therefore Ram kali will be able to send her daughter to school after 12 weeks.

61. $S_{10} = 210 \Rightarrow 5 [2a + 9d] = 210$

$$2a + 9d = 42$$

...(1)

$$S_{50} - S_{35} = 2565 \Rightarrow \frac{50}{2}[2a + 49d] - \frac{35}{2}[2a + 34d] = 2565$$

$$\frac{15}{2} (2a) + d [25 \times 49 - 35 \times 17] = 2565$$

$$15a + d [1225 - 595] = 2565$$

$$\text{or } 15a + 630d = 2565$$

$$\text{or } 3a + 126d = 513$$

...(2)

Solve (1) and (2) $d = 4, a = 3$.

62. $S_n = 5n^2 + 3n$

$$S_1 = a_1 = 8$$

$$S_2 = a_1 + a_2$$

$$26 = 8 + a_2 \Rightarrow a_2 = 18$$

$$d = 18 - 8 = 10$$

$$a_m = 168 \Rightarrow a + (m - 1)d = 168$$

$$8 + (m - 1)10 = 168 \Rightarrow m = 17$$

$$a_{20} = a + 19d = 8 + 190 = 198$$

63. $S_7 = 49, S_{17} = 289$ (Solve just like Q 53.)

64. $a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d$

$$a_{25} = a + 24d = -3d + 24d = 21d$$

$$a_{11} = a + 10d = -3d + 10d = 7d \quad a_{25} = 3a_{11}$$

65. Use $S_n = \frac{n}{2} [2a + (n - 1)d]$

Solve like in Q.53.

66. L.H.S. $= S_{12} = \frac{12}{2} [2a + 11d] = 6 [2a + 11d]$

$$\text{R.H.S.} = 3 \left[\frac{8}{2}(2a + 7d) - \frac{4}{2}(2a + 3d) \right] = 3[4a + 22d] = 6[2a + 11d]$$

Practice Test

Arithmetic Progression

Time: 1 Hr.

M.M. : 20

Section-A

1. Find the sum of first 10 natural numbers. 1
2. What is the common difference of an A.P. $8\frac{1}{8}, 8\frac{2}{8}, 8\frac{3}{8}, \dots$ 1
3. If $k, 2k - 1$ and $2k + 1$ are in A.P. then value of k is 1
4. The 10th term from the end of the AP $8, 10, 12, \dots, 126$ is 1

Section-B

5. How many 2 digit number are there in between 6 and 102 which are divisible by 6. 2
6. The sum of n terms of an A.P. is $n^2 + 3n$. Find its 20th term. 2
7. Find the sum $(-5) + (-8) + (-11) + \dots + (-230)$ 2

Section-C

8. Find the five terms of an A.P. whose sum is $12\frac{1}{2}$ and first and last term ratio is 2 : 3. 3
9. Find the middle term of an A.P. $20, 16, 12, \dots, -176$. 3

Section-D

10. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers. 4

□□□

Mathematics-X

65